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THESIS

AN ANALYSIS OF SENSOR EFFECTIVENESS TO INFORM A PREDICTIVE MAINTENANCE POLICY

by

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June 2009

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Joint Vision 2020 presents a plan for military dominance over the spectrum of military operations. One program that allows this to happen is Performance Logistics, which intends to increase availability and lower life cycle costs for weapon platforms. The ability to sense impending failures plays an important role in Performance Logistics. This thesis studies how sensor performance, as a tool of Condition Based Maintenance, affects the availability and cost of a generic component. Different types of maintenance policies are evaluated and compared using mathematical models. The maintenance protocols considered are *reactive* and *proactive*, namely: run to failure, scheduled inspection times, sensor based, and a combined inspection and sensor policy. Given parameters such as time and repair cost due to warnings or failures and frequency of inspection, it's found that a sensor influences the benefits of implementing a Condition Based Maintenance policy. In this thesis, results show improvement in availability and a reduced long-run average operating cost when the median of the random ratio of warning to failure time is roughly 0.8, the standard deviation is less than 0.1, and the mean time of maintenance for failure is greater than three times the mean time of repair due to warning.

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AN ANALYSIS OF SENSOR EFFECTIVENESS TO INFORM A PREDICTIVE MAINTENANCE POLICY

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Joint Vision 2020 presents a plan for military dominance over the spectrum of military operations. One program that allows this to happen is Performance Logistics, which intends to increase availability and lower life cycle costs for weapon platforms. The ability to sense impending failures plays an important role in Performance Logistics. This thesis studies how sensor performance, as a tool of Condition Based Maintenance, affects the availability and cost of a generic component. Different types of maintenance policies are evaluated and compared using mathematical models. The maintenance protocols considered are *reactive* and *proactive*, namely: run to failure, scheduled inspection times, sensor based, and a combined inspection and sensor policy. Given parameters such as time and repair cost due to warnings or failures and frequency of inspection, it's found that a sensor influences the benefits of implementing a Condition Based Maintenance policy. In this thesis, results show improvement in availability and a reduced long-run average operating cost when the median of the random ratio of warning to failure time is roughly 0.8, the standard deviation is less than 0.1, and the mean time of maintenance for failure is greater than three times the mean time of repair due to warning.

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THESIS DISCLAIMER

The computer code for this thesis has not been evaluated for all cases. Although every effort has been made, in the time allotted, to make sure the code is free from errors and logically correct it cannot be considered to have been thoroughly validated.

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EXECUTIVE SUMMARY

Joint Vision 2020 presents the plan for military dominance over the spectrum of military operations. One program that will allow this to happen is Performance Logistics. A component of this is Condition Based Maintenance Plus (CBM+):

> The CBM+ initiative covers a variety of technological and business changes designed to create a new maintenance environment in DOD. Projected changes focus on the vehicle platform with automated embedded sensor-based technologies providing standardized data in an integrated data environment. These initiatives include enhanced prognostics and diagnostic techniques, failure and trend analysis, electronic portable or point of maintenance aids, serial item management, Automated Identification Technology, and data-driven interactive training. (U.S. Department of Defense Under Secretary of Defense Acquisition, Technology, and Logistics, 2004)

The maintenance of weapon systems is expensive. In FY2007, the cost of DoD maintenance was more than $84 billion (2008 DoD Maintenance Factbook). Over the life of a weapon system, operations and support costs comprise roughly 65-80% of the total life cycle costs (Condition Based Maintenance Plus (CBM+); Department of Defense (DoD) Guidebook May 2008). Maintenance costs are typically a large percentage of those operational and support costs, potentially $67 billion in FY2007.

Currently, maintenance approaches are varied. Figure 1 displays properties of reactive and proactive maintenance approaches. Proactive categories consist of preventive and predictive. Predictive maintenance is divided into diagnostic and prognostic. Sensors may be used for predictive maintenance. The sensors are to warn of impending failures. For generality and analytic convenience, these predictive maintenance policies are studied using a failure time having an exponential distribution and a positively correlated warning time of impending failure. The ratio of warning time to failure time is a random variable independent of the failure time usually having a Weibull distribution. If

the ratio is greater than 1, then the warning does not occur and there is a failure without warning. The Weibull distribution is specified by its shape parameter and median. Upon replacement, repair or inspection during which the component is found failed the component is returned to a “as good as new” state. Renewal reward process theory is used to determine the operational availability and long-run average cost of a component for the different maintenance policies.

Maintenance Approaches				
	Reactive	Proactive		
Category	Run-to-fail	Preventive	Predictive	
Sub-Category	Fix when it breaks	Scheduled maintenance	Condition-based maint.-diagnostic	Condition-based maint.- prognostic
When Scheduled	No scheduled maintenance	Maintenance based on a fixed time schedule for inspect, repair and overhaul	Maintenance based on current condition	Maintenance based on forecast of remaining equipment life
Why Scheduled	N/A	Intolerable failure effect and it is possible to prevent the failure effect through a scheduled overhaul or replacement	Maintenance scheduled based on evidence of need	Maintenance need is projected as probable within mission time
How Scheduled	N/A	Based on the useful life of the component forecasted during design and updated through experience	Continuous collection of condition monitoring data	Forecasting of remaining equipment life based on actual stress loading
Kind of Prediction	None	None	On- and off-system, near-real-time trend analysis	On- and off-system, real-time trend analysis

Figure 1. Maintenance Approaches (From: U.S. Department of Defense Deputy Under Secretary of Defense for Logistics and Material Readiness, 2008)

The maintenance methods considered in the study are both reactive and proactive, namely; run to failure, scheduled inspection times, sensor based and, a combined inspection and sensor policy. The combined inspection and sensor policy can be thought of as a transitional policy where sensors for a sensor based policy are used in addition to the inspection of a component. Given parameters such as mean time and mean cost of inspections as well as repairs due to warnings or failures and frequency of inspection it is found that

implementing a predictive Condition Based Maintenance policy with a sensor can significantly improve the operational availability as well as reduce long-run average costs.

Two cases are considered. In the first case, failures are observable (the policies are run to failure with no sensor and condition based with sensor). In the second case, failures are not observable except by inspection or warning and the policies are inspection only or inspection in addition to possible sensor warning (scheduled inspection time and combined inspection and warning). The ability of the sensor to increase the operational availability of the component depends on the ratio of the mean repair time due to failure and that due to a warning; if the ratio is close to one, the sensor adds little value. It is also influenced by the variability of the ratio of sensor warning time to the time of failure; large variability can result in premature warning or no warning at all. Finally, it is influenced by the median of the ratio of the warning time to the failure; if the median of the ratio is close to one, then it is more likely the component will fail without warning; if the median of the ratio is small, then the warning tends to be premature.

For the parameters considered, results show sensor warnings improve availability and reduce long run average operating cost when the median of the random ratio of warning to failure time is 0.8 and the standard deviation of the ratio is less than 0.1; a sensor with these properties is called a good sensor. For a component with observable failures, if the mean repair time for failure is greater than three times the mean repair time for warning, operational availability increases by 3% to 4% with the sensor. Long run average costs are reduced by one half with a good sensor. For a component with non-observable failures, the sensor may have a greater beneficial effect on both operational availability and long run average cost since the sensor aids in eliminating component down time that occurs while waiting for inspection after the component has failed. For the parameter values considered, availability increases up to 25% and the long run average costs are reduced by a factor of five in some cases depending on the median of the ratio of the mean time to warning to the mean time to failure. The

presence of a sensor increases the inter-inspection time that maximizes the operational availability. If the ratio of the warning to failure time has a large variability, then the inter-inspection time maximizing the operational availability approaches that for a component with no sensor.

Effective sensors that allow Condition Based Maintenance prognostic and diagnostic maintenance policies to be successful have the potential to increase operational availability and reduce long run average costs. In situations where failures are observable, sensors can reduce catastrophic failures and the associated costs in time and money that go along with them by identifying a maintenance need based on the system's condition. In situations where observations are not observable and a component must be inspected to determine failure, a sensor can increase the inter-inspection time and warn of an impending failure eliminating the down time from when the component failed until the next scheduled inspection. These increased inter-inspection times and warnings reduce the time the component is down, reduce long run average costs and increase operational availability.

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I. INTRODUCTION: MAINTENANCE EFFICIENCY THROUGH FOCUSED LOGISTICS

A. BACKGROUND

As a part of Joint Vision 2020, Focused Logistics is one of the elements required to create a force that is dominant across the full spectrum of military operations (Joint Chiefs of Staff, 2000). The Focused Logistics transformation has three components, one of which is Force-centric Logistic Enterprise (FLE). Force-centric Logistic Enterprise itself is composed of six initiatives:

-Depot Maintenance Partnership

-Condition Based Maintenance Plus (CBM+)

-Total Life Cycle Systems Management (TLCSM)

-End to End Distribution

-Executive Agents

-Enterprise Integration

The DoD has proposed to instantiate FLE initiatives by leveraging commercial sector successes, to accelerate achievement of key Focused Logistics capabilities, such as agile sustainment and information fusion. (U.S. Department of Defense Under Secretary of Defense (Acquisition, Technology, and Logistics, 2004).

Of particular interest for this thesis is the CBM+ initiative.

> The CBM+ initiative covers a variety of technological and business changes designed to create a new maintenance environment in DOD. Projected changes focus on the vehicle platform with automated embedded sensor-based technologies providing standardized data in an integrated data environment. These initiatives include enhanced prognostics and diagnostic techniques, failure and trend analysis, electronic portable or point of maintenance aids, serial item management, Automated

> Identification Technology, and data-driven interactive training. (U.S. Department of Defense Under Secretary of Defense Acquisition, Technology, and Logistics, 2004)

CBM+ is an exciting and rapidly growing field. Each of the services has incorporated this system concept on some of its platforms. Each service names the maintenance policy uniquely but the same general methodology is behind each service's approach.

B. PURPOSE OF STUDY

With this background as framework, the following question is considered: How many, or what components of a system need to be embedded with sensor based technologies to make a CBM+ type maintenance approach more beneficial than a traditional maintenance approach? The objective of this thesis is limited to one component of a system, and investigates the effect of the quality of the sensor on operational availability and long run average cost. Sensitivity analysis determines the benefits of the CBM+ maintenance approach over a range of a sensor's detection ability.

C. ORGANIZATION OF STUDY

Chapter II gives additional general background of maintenance approaches, describing reactive and proactive categories. A short discussion of traditional and CBM+ types of maintenance practices is included. Chapter III considers two situations. In the first case, component failures are observable. In the second case component failure is not observable; the state of the component can be addressed by inspection or with the output of a sensor. Chapter III describes a mathematical model that represents the potential failure time of a component and a positively correlated warning time. Included is definition of times required for, and costs of, performing a given maintenance policy. Chapter IV applies the mathematical models developed in Chapter III to the different maintenance policies from Chapter II. Chapter V presents an analysis of results obtained from the models, followed with conclusions and recommendations.

II. MAINTENANCE COSTS ARE A LARGE PORTION OF DOD OPERATION AND SUPPORT (O&S) COSTS

A. O&S COSTS ARE A LARGE PORTION OF TOTAL LIFE CYCLE COSTS (LCC)

The maintenance of weapon systems is expensive. Over the life of a weapon system, operations and support costs comprise roughly 65-80% of the total life cycle costs (Condition Based Maintenance Plus (CBM+) Department of Defense (DoD) Guidebook May 2008). Maintenance costs are typically a large percentage of those operational and support costs. The cost of DoD maintenance in FY2007 was more than $84 billion (2008 DoD Maintenance Factbook). This maintenance work was conducted by more than 650,000 military and civilian maintainers who, along with several thousand commercial repair companies, supported 280 ships, 14,000 aircraft, 800 strategic missiles and 370,000 ground combat and tactical vehicles. The cost comes from both the training and support of personnel conducting the maintenance as well as the materiel required to replace or repair items that appear to be damaged or degraded.

B. MANY WAYS TO IMPLEMENT A MAINTENANCE POLICY

Maintenance approaches are varied. Figure 2 displays reactive and proactive maintenance categories. Proactive categories consist of preventive and predictive maintenance. Predictive maintenance is divided into diagnostic and prognostic.

Maintenance Approaches				
	Reactive	Proactive		
Category	Run-to-fail	Preventive	Predictive	
Sub-Category	Fix when it breaks	Scheduled maintenance	Condition-based maint.-diagnostic	Condition-based maint.- prognostic
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Figure 2. Maintenance Approaches (From: U.S. Department of Defense Deputy Under Secretary of Defense for Logistics and Material Readiness, 2008)

C. TRADITIONAL MAINTENANCE POLICIES ARE NOT ALWAYS EFFICIENT

Traditionally, maintenance approaches are categorized as reactive or proactive. Reactive (corrective) maintenance is conducted when a system fails. Proactive (preventive scheduled) maintenance is conducted according to a policy of inspections, repairs, and replacements based on component age. Reactive maintenance is performed on components selected to run to failure or on those that fail in an unplanned or unscheduled manner. Reactive maintenance is normally unscheduled since the failures occur unpredictably. Preventive maintenance can be based on calendar time, equipment-operating time or after a certain number of events or cycles. Preventive maintenance may be either scheduled or unscheduled. That is, preventive maintenance may be based on predetermined inspection time intervals or unscheduled after detection of a condition that may lead to failure or degraded performance of a system. A preventive policy of maintenance may result in inefficiencies by requiring the replacement or inspection of parts or components of systems prior to established need for inspection or replacement.

D. TECHNOLOGY CAN ENABLE IMPROVEMENTS IN MAINTENANCE EFFICIENCY

The use of technology potentially improves maintenance efficiency. The ability to monitor system and component operations without being intrusive, or degrading performance, has increased the use of predictive diagnostic maintenance practices. As of December 2007 DoD policy is: "Condition Based Maintenance Plus (CBM+) must be included in the selection of maintenance concepts, technologies, and processes for all new weapon systems, equipment and materiel programs based on readiness requirements, life-cycle cost goals and Reliability Centered Maintenance (RCM) based functional analysis" (U.S. Department of Defense, 2007). Maintenance that is more efficient will result in significant savings and reduced life cycle costs.

Condition based maintenance is a method of scheduling/conducting maintenance when needed, in time to prevent serious incapacitation or catastrophic failure of a system, and to improve system availability at lower cost. As defined in DoD Instruction 4151.22,

> CBM is a maintenance strategy based on equipment operational experience derived from analysis. CBM+ includes maintenance processes and capabilities derived from real-time or approximate real-time assessments of the condition of subsystems obtained from embedded sensors and/or external tests and measurements using either portable equipment or actual inspection. The objective of CBM+ is to suggest and diagnose maintenance need based on the evidence of need, while ensuring safety, reliability, availability, and reduced total ownership cost. (U.S. Department of Defense, 2007)

E. CBM+ LEVERAGES TECHNOLOGY TO MAKE MAINTENANCE MORE EFFICIENT

CBM+ uses technology to measure the condition of the components of a system. Components without monitoring sensors may need to be inspected periodically. The component is repaired when the sensor indicates failure is imminent or replaced when inspection indicates a failure has occurred. The use of sensors may decrease the number of inspections and preventive maintenance

time needed for the system. This practice invites the question: How many components of a system should be monitored and how does it affect the long run average time a system is up or functioning?

CBM+, when properly implemented, can reduce the need for inspections and age-based replacement or repair. Improper use of CBM+ results in overloading maintenance operations with unneeded repairs and replacements. Furthermore, infrequent sensor/inspection warnings fail to avert true failures. This suggests the possible existence of a “best” degree of monitoring.

This analysis examines the question:

How timely and accurate must sensors be to increase a component’s average operational availability? Satisfactory performance means a sensor’s ability to consistently predict failures in a timely manner: not too late and not too early.

The cost of the sensors and the component’s long run average up time or availability are studied as well as the long run average maintenance cost of the component. Gauthier (2006) discusses some attributes of a component that make it a candidate for sensor monitoring. Many important factors are identified.

-Sensing is preferred for a component that fails often vice rarely.

-Failure of the component should have relatively severe consequences.

-Direct physical inspection of a candidate component should be relatively difficult or resource intensive.

-A sensor must be feasible and appropriate for the component and not degrade the component’s performance.

-Time from warning to failure should be long enough to take corrective action.

-A sensor must be reliable and require minimum maintenance.

Implementation of CBM+ on components with these characteristics results in the best possibility of increasing availability and reducing operating cost.

F. METRICS HELP DEFINE AND EVALUATE APPROPRIATE APPLICATION OF A CBM+ POLICY

To determine the performance of CBM+, four metrics have been established by DoD as life cycle sustainment outcome metrics:

1. Materiel availability measures the percentage of total inventory of a system that is operationally capable of performing an assigned mission at a given time, based on material condition. One interpretation is the number of operationally capable end items divided by the total population. Materiel availability may also be defined as the percentage of time a system is operationally capable of performing an assigned mission, this measure can be expressed as the mean uptime of a system divided by the sum of mean system uptime and downtime; this measure is sometimes called operational availability, A_O.

2. Materiel reliability is the probability a system will perform without failure over a specific interval. Reliability must support a specified warfighting capability.

3. Ownership cost balances the sustainment solution by ensuring the O&S costs associated with materiel readiness. Cost Analysis Improvement Group's O&S Cost Estimating Structure supports this key system attribute (U.S. Department of Defense Cost Analysis Improvement Group, 1992).

4. Mean down time is the average time required to restore an asset to its full operational capabilities. Mean down time includes the time from the report of a down (not operational) asset until the time the asset returns to an operational state (U.S. Department of Defense Deputy Under Secretary of Defense for Logistics and Material Readiness, 2008).

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III. SYSTEM FAILURES AND WARNINGS CAN BE MODELED MATHEMATICALLY

A. TIMES TO FAILURES MODELED AS RANDOM VARIABLES

1. The Exponential Distribution

The exponential distribution is one probability model used to represent interarrival times between random events. The exponential distribution is not always the best distribution to model failure times but is often a convenient starting point. The exponential distribution has the property that it is "memoryless"; that is, the conditional distribution that a failure occurs within the next s time units is independent of the age of the system and so an age replacement policy is not useful.

For the scope of this thesis, failure times are exponentially distributed random variables that are independent and identically distributed having mean $\frac{1}{\lambda}(\lambda > 0)$. Since exponentially distributed random variables are memoryless, age replacement policies are not considered.

B. WARNINGS MODELED AS PREDICTIVE OF FAILURES

1. Sensors Warn of Impending Failure

Ideally, warnings of an impending failure occur before the failure happens and early enough to allow sufficient time to fix a problem before the failure leads to catastrophe. In modeling this behavior, consideration is given to the fact that the sensor will not always detect an impending failure and so the warning or the indication might occur too late. There are often many modes of failure that need to be considered in complex systems and numerous warnings for those different modes. In this thesis, the sensor's warning time will be positively correlated with the failure time. The warning time is modeled in the following manner:

Let $\boldsymbol{T}_F$ be an exponential random variable representing the time of failure and $\boldsymbol{T}_W$ be the random variable representing the time of the warning. We assume:

$$\boldsymbol{T}_W = \boldsymbol{\varphi} \boldsymbol{T}_F \tag{3.1}$$

where $\boldsymbol{\varphi}$ is a non-negative random variable independent of $\boldsymbol{T}_F$; note that $\boldsymbol{\varphi}$ is the ratio of the warning time to the failure time. This model allows for warning times to be a known fraction of failure time if $\boldsymbol{\varphi}$ is a constant between zero and one. For example, if $\boldsymbol{\varphi}$ is equal to one, then the warning signal would occur as the failure was occurring. The model allows for the possibility of failures to occur without a warning; if $\boldsymbol{\varphi}$ is greater than one then the component fails without warning. While it is not likely, in reality, that a warning would go off after a plane has crashed, this model representation does allow for modeling the absence of a warning prior to failure due to a poor sensor. For this thesis the ratio of warning to failure time, $\boldsymbol{\varphi}$, is an independent Weibull random variable with scale parameter α and shape parameter β where the Weibull distribution probability density function (pdf) is defined as:

$$g(\alpha,\beta,t) = \begin{cases} 0 & t < 0 \\ \alpha\beta\alpha^{\beta-1}t^{\beta-1}e^{-(\alpha t)^{\beta}} & t \geq 0 \end{cases} \tag{3.2}$$

The cumulative distribution function (cdf) for the Weibull is:

$$G(\alpha,\beta,t) = \begin{cases} 0 & t < 0 \\ 1 - e^{-(\alpha t)^{\beta}} & t \geq 0 \end{cases} \tag{3.3}$$

The parameters of the Weibull distribution are specified by the shape parameter and the median. A large range of sensor performance can be investigated utilizing the versatility of the shape of the Weibull distribution by choosing α and β appropriately. Appendix B displays selected graphs of Weibull pdfs as well as the equations for the mean, median and variance. The covariance of $\boldsymbol{T}_F, \boldsymbol{T}_W$ is also included for each of the parameters α, β used in the thesis.

C. TIME AND COSTS FOR COMPONENT FAILURES, WARNINGS AND, REPLACEMENT OR REPAIR

For the scope of the thesis, consideration is given to the time and expense of inspection, repair, and replacement of a system component. These times and costs are considered as input variables to determine of the effectiveness of instituting a proactive CBM+ type of maintenance for a given component with a parameter defined sensor. Table 1 describes the input variables.

Parameter	Description
r_I	Expected Time required to conduct scheduled inspection
r_F	Expected Time required to conduct maintenance due to a failure
r_W	Expected Time required to conduct maintenance due to a warning
c_I	Expected Cost to conduct a scheduled inspection
c_F	Expected Cost required to conduct maintenance due to a failure
c_W	Expected Cost required to conduct maintenance due to a warning
$\boldsymbol{T}_F$	Random variable having an exponential distribution representing a time of failure
$\boldsymbol{T}_W$	Random variable representing a time of warning

Table 1. Time and Cost Parameters

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IV. MODEL CHOICES FOR MAINTENANCE

A. CYCLE TIMES

Renewal reward processes are used to compute operational availability and long run average costs for different maintenance policies. The renewal reward process allows for conveniently computing these values. “For a cycle that is completed every time a renewal occurs, the long run average reward per unit time is equal to the expected reward earned during a cycle divided by the expected length of a cycle” (Ross, 2007). *For* determining the component operational availability of different maintenance policies, the reward is the time the system is up during a cycle and the cycle time is that time between the inspections, warnings or failures depending on the model.

B. MAINTENANCE MODELS DESCRIBED MATHEMATICALLY WHEN FAILURES ARE OBSERVABLE

1. Run to Failure Model (RTF)

The RTF model is a traditional maintenance policy model. It is assumed failures of the component are observable. The run to failure cycle length is the time between failures. The operational availability is simply the mean amount of time the component is up divided by the sum of mean up and mean down times. Figure 3 displays this graphically while equation (4.1) describes this model mathematically. Equation (4.2) displays the expression for the long run average cost.

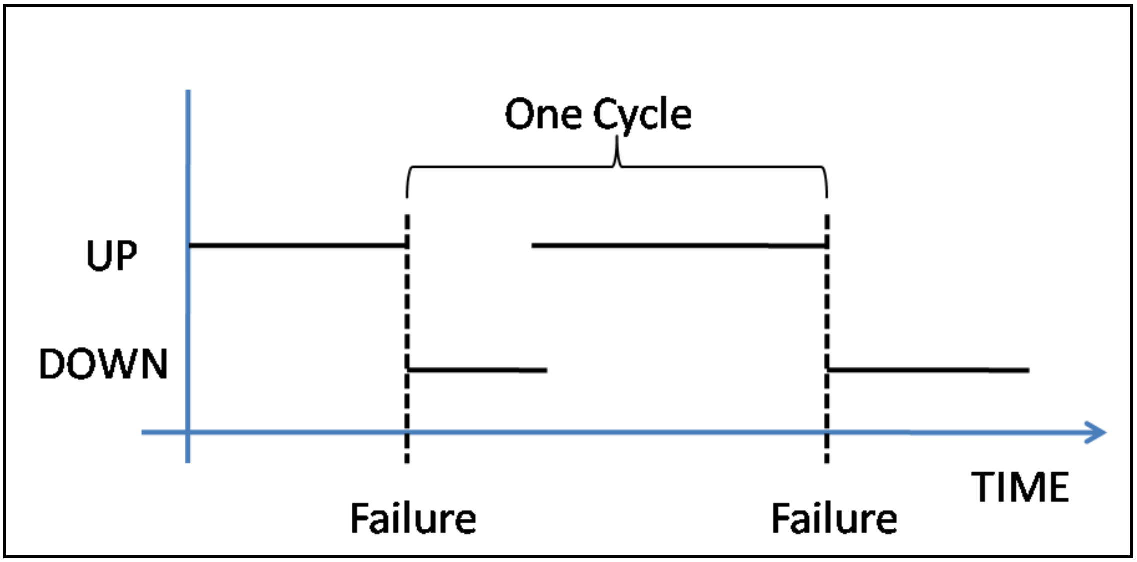

Figure 3. Graphical Depiction of Run to Failure Model

$$\frac{E[UpTime]}{E[CycleTime]} = \frac{E[T_F]}{E[T_F] + r_F} \quad (4.1)$$

$$E[Long\ Run\ Average\ Cost] = \frac{c_F}{E[CycleTime]} \quad (4.2)$$

2. Sensor Based Predictive Model (Sensors)

The sensor only model incorporates only warnings and failures to define its cycle times. Failures are observable. A graphical depiction of this model is Figure 4. The up time is the minimum of the time until a warning or failure. The cycle time is that minimum time plus the time due to repair from a warning or a failure depending on which event occurs. Equations (4.3) and (4.4) display operational availability and long run average cost equations respectively. Further details on the derivation of these equations are found in Appendix C.

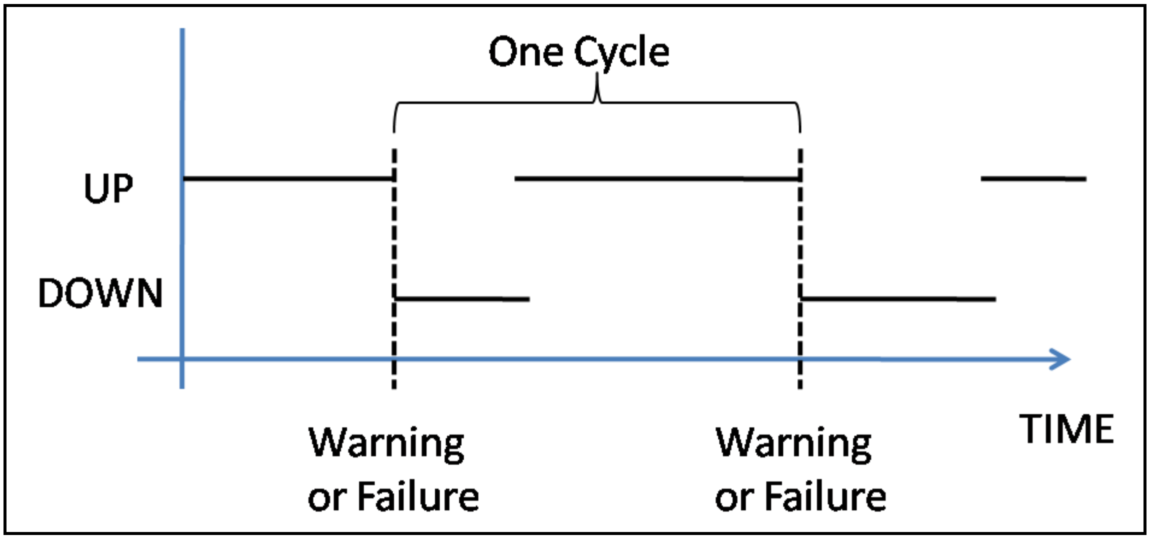

Figure 4. Graphical Depiction of Sensor Based Maintenance Model

$$\frac{E[UpTime]}{E[CycleTime]} = \frac{E[\min(\boldsymbol{T}_W, \boldsymbol{T}_F)]}{E[\boldsymbol{R}] + E[\min(\boldsymbol{T}_W, \boldsymbol{T}_F)]} \tag{4.3}$$

$$E[Long\ Run\ Average\ Cost] = \frac{c_F e^{-\alpha^\beta} + c_W\left[1 - e^{-\alpha^\beta}\right]}{E[CycleTime]} \tag{4.4}$$

where $E[\boldsymbol{R}]$ is the expected time of repair and is expressed as follows:

$$E[\boldsymbol{R}] = P\{\boldsymbol{T}_W < \boldsymbol{T}_F\} r_W + P\{\boldsymbol{T}_F \le \boldsymbol{T}_W\} r_F = \left[1 - e^{-\alpha^\beta}\right] r_W + r_F e^{-\alpha^\beta} \tag{4.5}$$

C. MAINTENANCE MODELS FOR A COMPONENT WITH NON-OBSERVABLE FAILURES

1. Scheduled Maintenance Model (Inspections)

In this model, the failures of the component are not observable. The condition of the component can only be determined by inspection. When the component fails, the component is down until its next inspection and a working component is down during its inspection. Since the lifetime of the component has an exponential distribution, the Scheduled Maintenance model has cycle

times that are a sum of the inter-inspection time, T, and repair time. A graphical depiction of the situation is Figure 5. The mathematical expressions for operational availability and long run average cost are given in equations (4.5) and (4.6) respectively. The inspections will find the system in an operating or failed state and repair as required. For the purpose of this thesis, failures resulting from inspection are not considered. This assumption makes it a best-case scenario for comparison against other maintenance methods. Appendix D presents the derivation of equations (4.5) and (4.6).

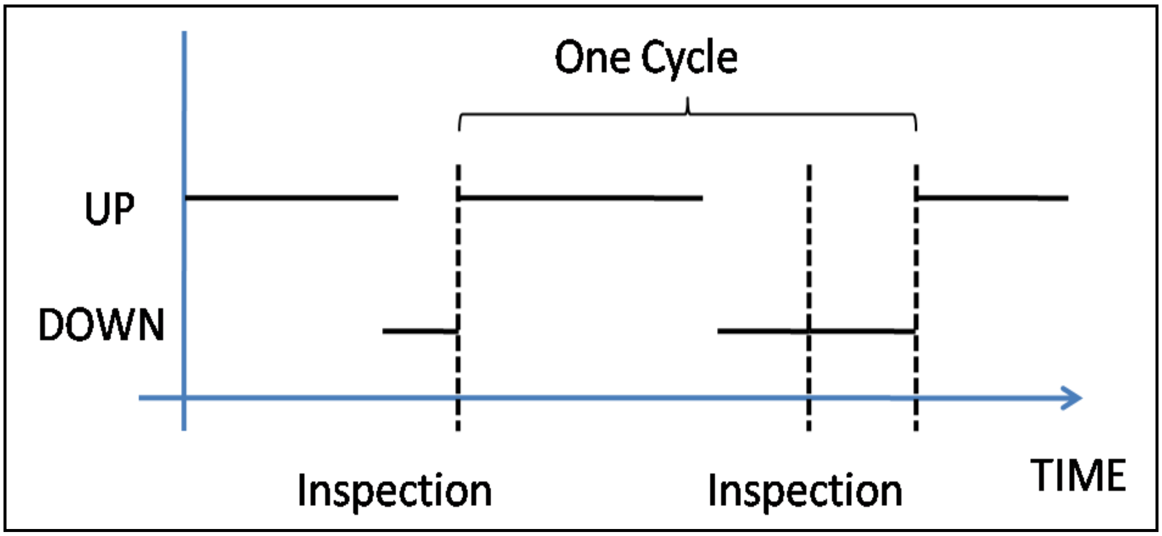

Figure 5. Graphical Depiction of Scheduled Maintenance Model

$$\frac{E[UpTime]}{E[CycleTime]} = \frac{\frac{1}{\lambda}\left[1-e^{-\lambda T}\right]}{r_I + r_F\left[1-e^{-\lambda T}\right]+T} \tag{4.5}$$

$$E[Long\ Run\ Average\ Cost] = \frac{c_I + c_F\left[1-e^{-\lambda T}\right]}{E[CycleTime]} \tag{4.6}$$

For this model, an optimal inspection time that maximizes operational availability can be calculated by taking the derivative with respect to T of (4.5), a concave function, setting it equal to zero, and solving numerically. The optimal inspection time can also be approximated using a Taylor series expansion,

$e^{-\lambda T} \approx 1-\lambda T$ and solving (4.5) for T, the result appears in equation (4.7). The approximation is described in more detail in Appendix C.

$$T^*_{approx} = \frac{-\lambda r_I + \sqrt{(\lambda r_I)^2 + 4\lambda r_I}}{2\lambda} \tag{4.7}$$

Results from this approximation compared to the numerically calculated values are in Table 2 and displayed graphically in Figure 6 for various values of the mean time to conduct an inspection r_I; the mean repair time for a failure, $r_F = 2$, for all cases. Due to the flat nature of the availability curve, the optimal T approximations are low while the operational availability values are in close proximity. The inter-inspection time that maximizes the operational availability is found by search over the time interval from 0 to 300 with a grid of one time unit.

For $r_F = 2$	**Numerical Result**		**Approximation**	
	Optimal T	A_O	Optimal T	A_O
$r_I = 1$	14	0.86	9.5	0.85
$r_I = 2$	19	0.81	13.2	0.80
$r_I = 3$	24	0.78	15.9	0.77
$r_I = 4$	27	0.75	18.1	0.74

Table 2. Optimal Inter-Inspection Intervals and Approximations for Given r_I

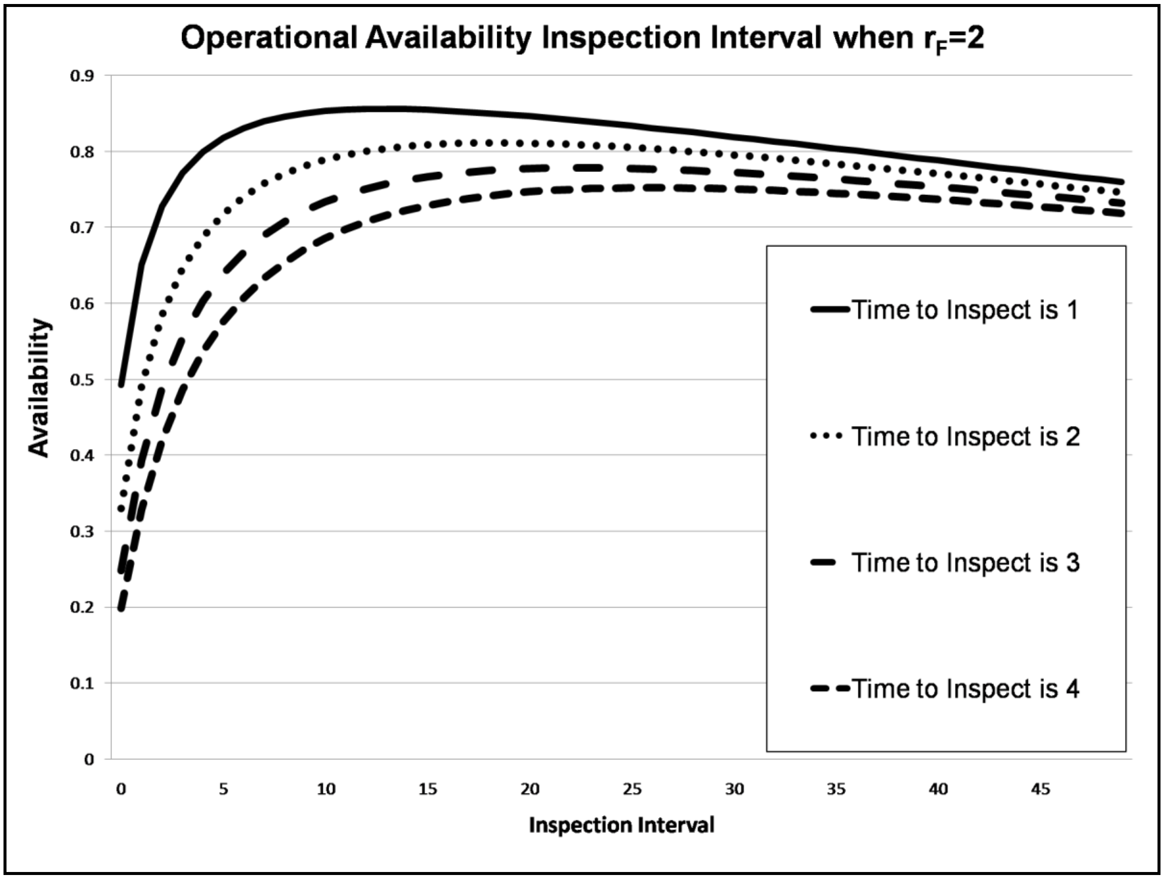

Figure 6. Plot of Availability for Given Inter-Inspection Intervals

2. Inspection and Sensor Predictive Model (Inspections & Sensors)

Although not a purely predictive model, this model with inspections and sensor is a transitional type of maintenance that might be used as a purely predictive maintenance policy is implemented. It is assumed that failures are not directly observable and can only be detected by inspection. For this case, the sensor may provide warning of impending failure. The system also is inspected at fixed intervals of time, T. The component is repaired to as good as new whenever a warning occurs or when the system is found failed during an inspection. When the component fails without warning the component is down until its next inspection. Figure 7 represents this model. Equations (4.8) and (4.9) display the operational availability and long run average cost respectively; G is the distribution of the ratio of the warning to repair time, φ. Appendix E

gives details of the derivation of (4.8) and (4.9). The integrals are evaluated numerically using Gaussian quadrature with 24 points (Abramowitz M., 1972).

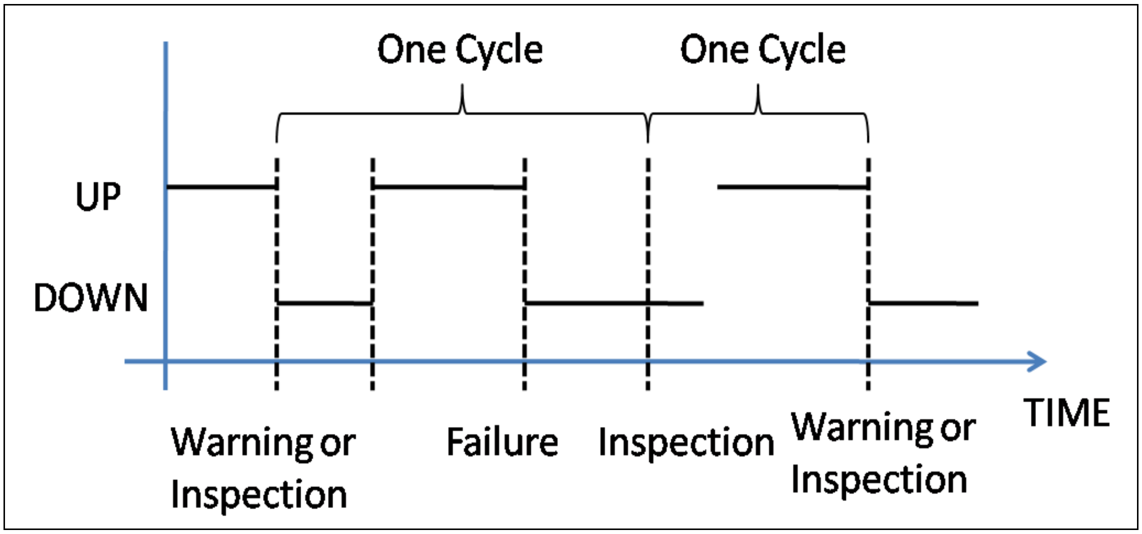

Figure 7. Graphical Depiction of Inspection and Sensor Model

$$\frac{E\left[UpTime\right]}{E\left[Cycle\,Time\right]} = \tag{4.8}$$

$$\frac{E\left[\boldsymbol{T}_F\right]E\left[\min(\boldsymbol{\varphi},1)\right]}{P\{\boldsymbol{\varphi}>1\}\left[\frac{1}{(1-e^{-\lambda T})}(r_I+T)+r_F\right]+P\{\boldsymbol{\varphi}<1\}r_W+r_I\int_0^1\frac{1}{e^{(\lambda/y)T}-1}G(dy)+E\left[\boldsymbol{T}_F\right]\int_0^1 yG(dy)}$$

$$E\left[Long\ Run\ Average\,Cost\right] = \tag{4.9}$$

$$\frac{c_F e^{-\alpha^\beta}+c_W\left[1-e^{-\alpha^\beta}\right]+c_I\int_0^1\frac{e^{-(\lambda/y)T}}{1-e^{-(\lambda/y)T}}G(dy)+c_I e^{-\alpha^\beta}\left[\frac{1}{1-e^{-\lambda T}}\right]}{E\left[Cycle\,Time\right]}$$

D. COMMON FACTORS FOR TRADITIONAL AND CBM+ MAINTENANCE PROCESSES

For each of the maintenance policies, measures described in Chapter II determine the metrics that evaluate the efficiency of the maintenance that is performed. For the scope of this thesis, factors considered are the time between inspections or inter-inspection time, the expected time to conduct an inspection, the expected repair time due to warning, and the expected repair time due to failure. Similarly, the cost factors are expected costs of inspection, of repair due to warning and of repair due to failure. In the following numerical examples, the component failure rate is $\lambda = 0.01$ which for an exponential distribution corresponds to a Mean Time Between Failure (MTBF) of $\left(\frac{1}{\lambda}\right) = 100$ time units. The expected cost or repair time required due to a warning is fixed and equal to 2 for all cases. Failure and inspection expected costs and times are varied with values displayed in Table 3. Since the point of a CBM+ type maintenance policy is to reduce cost and increase availability, consideration is not given to cases where it is more expensive to warn than to fail. This thesis also assumes that there is no cost for the sensor; this is an unrealistic assumption, which is addressed in the conclusions and recommendations section. However, expected inspection costs both more and less expensive than expected repair costs due to warnings are considered.

	Mean Repair Time and Cost Values (for these cases the values will be equal)		
Cases Considered	**Inspection**	**Warning**	**Failure**
1	1	2	2
2	1	2	3
3	1	2	4
4	1	2	5
5	1	2	6
6	1	2	7
7	2	2	2
8	2	2	3
9	2	2	4
10	2	2	5
11	2	2	6
12	2	2	7
13	3	2	2
14	3	2	3
15	3	2	4
16	3	2	5
17	3	2	6
18	3	2	7
19	4	2	2
20	4	2	3
21	4	2	4
22	4	2	5
23	4	2	6
24	4	2	7

Table 3. Common Cost and Time Parameters for Calculations

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V. ANALYSIS OF MODEL RESULTS: CBM+ CAN BE BENEFICIAL WHEN APPLIED APPROPRIATELY

A. CBM+ SHOWS IMPROVEMENT OVER TRADITIONAL REACTIVE AND PREVENTIVE MAINTENANCE BASED ON AVAILABILITY MEASURE OF PERFORMANCE (MOP)

The values displayed in the tables of this chapter are a selection of model results from Appendix A. In Appendix A are the results computed using the equations in Chapter IV for each of the maintenance polices for each of the cases listed in Table 3 and the parameters for the Weibull random variable as listed in Appendix F. For those maintenance policies that include inspection, the inter-inspection time that maximizes the operational availability is found by search over the time interval from 0 to 300 with a grid of one time unit. Integrals are evaluated numerically using Gaussian quadrature with 24 points.

Table 4 displays results for case 5 from Table 3; in this case, there is a large expected time to repair due to failure and a small expected time to conduct inspections. The Weibull random variable with shape parameter 10 is considered at three different values of the median of the ratio of warning to failure times: 0.5, 0.8, and 0.9. These values represent a consistent sensor since the variance is quite low, approximately 0.01. Appendix B, Figure 13 displays the density function for these Weibull distributions with shape parameter 10. Two scenarios are displayed in Table 4: in one, the failure is observable and in the other the failure is not observable. For the case in which failure is observable, an operational availability of 94% results from a traditional run to failure maintenance policy. The presence of a sensor increases the availability 2% to 3%. In this case the sensor maintenance policy attains 2% to 3% higher availability depending on the sensor's ability to detect impending failures consistently for the given median value.

When failures are not observable, the policy of inspection and sensor attains at least a 9% higher availability than inspection alone. Of note here is the

fact that when the median of the ratio of warning to failure times is 0.5 the probability a warning comes before failure is 1, so the optimal inspection policy with the sensor is to never inspect. Note that the presence of the sensor increases the inter-inspection time that maximizes the operational availability. The decrease in operational availability when the median ratio of warning to failure times is = 0.9 is the result of warnings not occurring before failure and the resulting down time between the failure time and the next inspection.

$\beta = 10$	Median of ratio of warning to failure time= 0.5		Median of ratio of warning to failure time= 0.8		Median of ratio of warning to failure time= 0.9	
	% Avail	Optimal Inter-Inspection Time	% Avail	Optimal Inter-Inspection Time	% Avail	Optimal Inter-Inspection Time
	Failure Observable					
Run To Failure	0.94	N/A	0.94	N/A	0.94	N/A
Sensor	0.96	N/A	0.97	N/A	0.97	N/A
	Failure Not Observable					
Inspections	0.83	14	0.83	14	0.83	14
Inspections & Sensor	.96	∞	0.97	242	0.92	34
$P\{\boldsymbol{T}_W < \boldsymbol{T}_F\}$	1.00		0.999		0.578	

Table 4. Availability for Case 5 when Shape Parameter is 10

Again, consider the shape parameter equal to 10 with the three different values for the median of the ratio of warning to failure times. Table 5 displays results for Case 21 from Table 3 when the mean time to repair due to failure as well as the mean time to conduct an inspection is twice that of mean repair time due to warning. For the case in which failures are observable the maximum improvement achieved with a sensor is about a one and a half percent improvement over the traditional run to failure policy. The policies for a component whose failures are not observable can give up to a 20% increase in availability. Here the benefit of the sensor is greater because the sensor may give a warning to indicate impending failure and so eliminates the down time from the failure of the component until the next inspection. Note that the presence of a sensor increases the inter-inspection time that maximizes the operational availability. Comparison with the results of Table 5 suggests that the increase in the mean time to conduct an inspection has resulted in the increase of the maximizing inter-inspection times.

$\beta = 10$	Median of ratio of warning to failure time= 0.5		Median of ratio of warning to failure time= 0.8		Median of ratio of warning to failure time= 0.9	
	% Avail	Optimal Inter-Inspection Time	% Avail	Optimal Inter-Inspection Time	% Avail	Optimal Inter-Inspection Time
	Failure Observable					
Run To Failure	0.96	N/A	0.96	N/A	0.96	N/A
Sensor	0.96	N/A	0.975	N/A	0.975	N/A
	Failure Not Observable					
Inspections	0.74	27	0.74	27	0.84	27
Inspections & Sensor	.96*	∞	0.971	242	0.89	64
$P\{\boldsymbol{T}_W < \boldsymbol{T}_F\}$	1.00		0.999		0.578	

Table 5. Availability for Case 21 when Shape Parameter is 10

Additional investigation is conducted using a model of a sensor that has more variation in the ratio of the warning to the failure time as represented by a Weibull distribution with shape parameter 0.25. Table 6 displays the operational availabilities of the different maintenance policies for case 5, where mean time to conduct an inspection is 1/6th that of the mean repair time due to failure; the mean repair time due to warning is 1/3rd that of a mean repair time due to failure. Comparison with Tables 4 and 5 for a sensor with a shape parameter of 10 suggests there is nothing gained by adding a sensor to the component when

failure is observable; in fact, there is a decrease in operational availability presumably due to premature warnings. When failures are not observable the component availability with a sensor is approximately the same as without a sensor. Further the maximizing inter-inspection intervals are about the same with and without a sensor.

$\beta = 0.25$	**Median of ratio of warning to failure time= 0.5**		**Median of ratio of warning to failure time= 0.8**		**Median of ratio of warning to failure time= 0.9**	
	A_O	Optimal Inter-Inspection Time	A_O	Optimal Inter-Inspection Time	A_O	Optimal Inter-Inspection Time
	Failure Observable					
Run To Failure	0.943	N/A	0.943	N/A	0.943	N/A
Sensor	0.933	N/A	0.935	N/A	0.935	N/A
	Failure Not Observable					
Inspections	0.828	14	0.828	14	0.828	14
Inspections & Sensor	0.83*	15	0.83	15	0.83	15
$P\{\boldsymbol{T}_W < \boldsymbol{T}_F\}$	0.56		0.52		0.51	

Table 6. Availability for Case 5 when Shape Parameter is 0.25

Table 7 displays the operational availabilities of the different maintenance policies for case 21 when the shape parameter is 0.25; the mean time to conduct

an inspection is equal to the mean repair time due to failure; and the mean repair time due to warning is half that of a mean repair time due to failure. Results here show that when a sensor is utilized the operational availability decreases when failures are observable. When the failures are not observable, the operational availability increases marginally; in this case about 1%.

β = 0.25	**Median of ratio of warning to failure time= 0.5**		**Median of ratio of warning to failure time= 0.8**		**Median of ratio of warning to failure time= 0.9**	
	A_o	Optimal Inter-Inspection Time	A_o	Optimal Inter-Inspection Time	A_o	Optimal Inter-Inspection Time
	Failure Observable					
Run To Failure	0.962	N/A	0.962	N/A	0.962	N/A
Sensor	0.948	N/A	0.950	N/A	0.950	N/A
	Failure Not Observable					
Inspections	0.741	27	0.741	27	0.741	27
Inspections & Sensor	0.751	29	0.751	29	0.750	29
$P\{\boldsymbol{T}_W < \boldsymbol{T}_F\}$	0.56		0.52		0.51	

Table 7. Availability for Case 21 when Shape Parameter is 0.25

B. COMPARISON OF ANALYTICAL RESULTS

For this section, we assume the component's failure is not observable. Table 8 displays operational availabilities for two cases; one in which the sensor ratio of warning to failure time, φ, is constant and the other in which φ has a Weibull distribution with a large shape parameter and median equal to the constant.

To ensure the results of the stochastic model are reasonable for the maintenance policy that includes both Sensor and Inspection two models are compared. In one model the probability φ equals 0.9 (respectively 0.5) is 1. The operational availability is compared to that of a model in which φ has a Weibull distribution with a median of 0.9 and a shape parameter of 30 (respectively a median of 0.5 and shape parameter of 5). A sample of the results for given inter-inspection intervals, T, are displayed in Table 8 below; the values over the rest of the considered range are similar in their proximity. The numbers are similar because of equal medians and the large shape parameter values. Additional results can be found in Table 29, Appendix G.

	Constant φ	φ having a Weibull Distribution	Constant φ	φ having a Weibull Distribution
Inspection Time T	P{$\varphi=.9$}=1	Median =0.9 and Shape Parameter = 30	P{$\varphi=.5$}=1	Median =0.5 and Shape Parameter = 5
1	0.49587	0.49584	0.49260	0.49251
2	0.65933	0.65929	0.65357	0.65341
3	0.74072	0.74067	0.73344	0.73324
4	0.78945	0.78938	0.78117	0.78094
5	0.82188	0.82181	0.81290	0.81265
6	0.84503	0.84495	0.83551	0.83525

Table 8. Operational Availability

C. CBM+ MAINTENANCE METHODS DECREASE COSTS

The use of a sensor to detect impending failures has the opportunity to decrease costs when failures are observed and even more so when failures can't be observed. The inter-inspection time that minimizes the long run average cost is found by search over the time interval from 0 to 300 with a grid of one time unit. Integrals are evaluated numerically using Gaussian quadrature with 24 points.

The first example is case 3 of Table 3 with shape parameter 10; the values are displayed in Table 9. The long run average cost (LRAC) for policies when failures are observable is 1 minus the availability since the mean costs are the same as the mean times. This is not true for the policies for components whose failures are not observable. We first discuss the case when failures are

observable. For this example, when the median ratio of warning time to failure time is 0.8 or 0.9, the long run average cost of Run to Failure is roughly 1.5 times more than that with a sensor, (0.038/0.025 ~ 1.5). When the median ratio is 0.5 the long run average costs are about the same as that for the component without a sensor. For the case in which failures are not observable, the long run average cost of an inspection policy without a sensor can be two to three times more than that with a sensor. The presence of a sensor increases the inter-inspection interval that minimizes long run average cost.

$\beta = 10$	**Median of ratio of warning to failure time= 0.5**		**Median of ratio of warning to failure time= 0.8**		**Median of ratio of warning to failure time= 0.9**	
	LRAC	Optimal Inter-Inspection Time	LRAC	Optimal Inter-Inspection Time	LRAC	Optimal Inter-Inspection Time
	Failure Observable					
Run To Failure	0.038	N/A	0.038	N/A	0.038	N/A
Sensor	0.039	N/A	0.025	N/A	0.025	N/A
	Failure Not Observable					
Inspections	0.098	14	0.098	14	0.098	14
Inspections & Sensor	0.039	∞	0.026	197	0.048	34

Table 9. Long Run Average Costs for Case 3 when Shape Parameter is 10

Table 10 displays a second example of case 3 of Table 3 when the shape parameter is 0.25. For the case when failures are observable, the cost of Run to Failure is roughly 3/4 the cost of the sensor policy, (0.038/0.052 ~ 0.73) for all medians; this is the result of premature failures. With non-observable failures, this case shows that the long run average cost of an inspection policy without a sensor is about 95% of that with a sensor (0.098/0.103 ~ 0.95). The minimizing inter-inspection times are about the same with and without a sensor. A sensor whose warning time has this much variability negligibly decreases the long run average cost.

$\beta = 0.25$	**Median of ratio of warning to failure time= 0.5**		**Median of ratio of warning to failure time= 0.8**		**Median of ratio of warning to failure time= 0.9**	
	LRAC	Optimal Inter-Inspection Time	LRAC	Optimal Inter-Inspection Time	LRAC	Optimal Inter-Inspection Time
	Failure Observable					
Run To Failure	0.038	N/A	0.038	N/A	0.038	N/A
Sensor	0.052	N/A	0.050	N/A	0.050	N/A
	Failure Not Observable					
Inspections	0.098	14	0.098	14	0.098	14
Inspections & Sensor	0.104	15	0.103	15	0.102	15

Table 10. Long Run Average Costs for Case 3 when Shape Parameter is 0.25

D. SENSOR PARAMETER CHARACTERISTICS AND PREVENTING COMPONENT FAILURE DURING A MISSION

Another measure of performance for the sensor is how much time elapses between the warning and the failure. If a warning occurs too soon, extra maintenance is required; however, if it occurs too late the failure may occur before the mission is complete. A simulation to determine the effect of the sensor on completing missions without failure was conducted; details of the simulation appear in Appendix F. The Weibull distribution for the warning time has parameter values listed in Table 11. The length of a mission is 2 time units throughout. The number of replications is 10,000. The simulation utilizes the Mersenne Twister random number generator (Matsumoto M., 2003). The third column of Table 11 displays the fraction of missions with a warning that also include a failure. The fourth column displays the standard error of the third column. The fifth column displays the average number of missions without failure after the warning not to include the mission having the failure or the mission with the warning. The last column is the standard error of this average.

Shape of Weibull	Median of Weibull	Fraction of Missions with Failure Given a Warning Occurs During the Mission	Standard Error of Fraction of Missions with Failure Given Warning Occurs During the Mission	Average # Missions Post Warning without Failure	Standard Error of Average # Missions Post Warning without Failure
0.25	0.5	0.038	0.002	11.6	0.0007
	0.8	0.042	0.002	10.3	0.0006
	0.9	0.042	0.002	10.0	0.0006
0.5	0.5	0.035	0.002	10.7	0.0005
	0.8	0.042	0.002	8.3	0.0004
	0.9	0.043	0.002	7.8	0.0004
1.0	0.5	0.03	0.002	10.2	0.0005
	0.8	0.04	0.002	6.1	0.0003
	0.9	0.05	0.002	5.4	0.0003
2	0.5	0.02	0.001	12.2	0.0006
	0.8	0.04	0.002	4.5	0.0002
	0.9	0.05	0.002	3.4	0.002
5	0.5	0.02	0.001	19.4	0.001
	0.8	0.03	0.001	3.9	0.0001
	0.9	0.05	0.002	1.8	0.0001
10	0.5	0.02	0.001	21.8	0.002
	0.8	0.03	0.001	5.6	0.0003
	0.9	0.06	0.002	1.5	0.00001

Table 11. Sensor Effect on Mission Times of Length Two

The importance of the median of the ratio of the warning time to the failure time is apparent from the results of Table 11. For example, when the shape parameter is 10 and median of the ratio is 0.9, then 6% of the missions with a warning will have a failure in the same mission; while if the median ratio is 0.5 then there is only about a 2% probability of a failure occurring during a mission that has a warning. For a fixed Weibull shape parameter, the fraction of missions with a warning that also have a failure is nondecreasing as the median of the ratio of warning to failure time increases; the mean number of additional missions without failure after the warning decreases as the median of the ratio increases. For fixed median of the ratio the mean number of additional missions without failure after the warning decreases as the shape parameter increases.

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VI. CONCLUSIONS: HOW MUCH BETTER CAN CBM+ BE THAN TRADITIONAL MAINTENANCE?

A. CBM+ AND PREVENTIVE MAINTENANCE POLICIES

Preventive maintenance can be both cost effective and improve the operational availability of components when properly applied. However, if the sensor tends to give premature warnings or no warning at all, then the sensor use will decrease operational availability and increase long run average maintenance costs.

We first discuss the case of observable failures. Table 12 displays 1 minus the ratio of the component's operational availability with and without a sensor. The parameters of the Weibull distribution are along the left side and the cases, from Table 3, being considered are across the top. When failures are not observed the inspection times/costs do not contribute to operational availability; hence the multiple cases listed at the top of each column. Negative values result when the operational availability of the component with a sensor is less than that without a sensor; this can occur when the shape parameter of the ratio of warning to failure time distribution is small which results in greater warning time variability with resulting premature warnings and failures without warnings. Results presented in Tables 4 through 7 and Appendix A suggests that when failures are observable, three parameters influence the use of sensor warning times to increase operational availability. One parameter is the ratio of the mean repair time due to failure and that due to warning; if the ratio is close to one, then the sensor adds little value. The second parameter is the median of the Weibull distribution of the ratio of warning to failure time; the closer the median is to one, the more likely that the component will fail without warning. The third parameter is the shape parameter of the Weibull distribution of the ratio of warning to failure time; this parameter influences the variability of the warning time; large variability can result in premature warning or no warning at all. For the parameter values presented, the addition of a sensor increases the operational availability when

the mean repair time for failures is greater than three times that for warnings; the shape parameter of the Weibull distribution for the ratio of warning to failure is greater than 2; and the Weibull median is either 0.8 or 0.9; we call a sensor with these properties a good sensor.

From the data developed in the previous chapter consideration for using sensors on components when failures are observable should be given when the time of repair is greater than three times that which results from a warning. If a sensor can predict better than roughly 80% of failures with a median ratio of the warning to failure time about 80%, then a sensor should be considered. For a good sensor, if the mean cost of repair due to failure is greater than three times the mean repair cost due to a warning, then the run to failure policy can be as much as twice as expensive. The values for long run average costs are the same as 1 minus the operational availability since the same values of time and cost were used, Table 24, Appendix A displays the ratios of the long run average costs for these two maintenance methods.

Comparison of Sensor and Run to Failure Maintenance Policies on Availability (1-(RTF / Sensor))

		Case Numbers for Time and Cost Parameters					
Shape	Median	1,7,13,19	2,8,14,20	3,9,15,21	4,10,16,22	5,11,17,23	6,12,18,24
0.25	0.5	-0.018	-0.016	-0.015	-0.013	-0.011	-0.010
	0.8	-0.015	-0.014	-0.012	-0.011	-0.009	-0.008
	0.9	-0.015	-0.013	-0.012	-0.010	-0.009	-0.008
0.5	0.5	-0.017	-0.014	-0.011	-0.008	-0.005	-0.002
	0.8	-0.013	-0.010	-0.008	-0.005	-0.003	-0.001
	0.9	-0.012	-0.009	-0.007	-0.005	-0.003	-0.001
1	0.5	-0.017	-0.011	-0.006	-0.001	0.004	0.009
	0.8	-0.010	-0.006	-0.002	0.001	0.005	0.008
	0.9	-0.009	-0.005	-0.002	0.001	0.004	0.008
2	0.5	-0.018	-0.009	-0.001	0.008	0.016	0.024
	0.8	-0.007	-0.002	0.003	0.008	0.013	0.018
	0.9	-0.006	-0.001	0.003	0.007	0.012	0.016
5	0.5	-0.020	-0.010	0.000	0.009	0.018	0.028
	0.8	-0.005	0.003	0.011	0.019	0.027	0.034
	0.9	-0.003	0.003	0.009	0.015	0.021	0.027
10	0.5	-0.020	-0.010	-0.001	0.009	0.018	0.028
	0.8	-0.005	0.005	0.014	0.023	0.033	0.042
	0.9	-0.003	0.006	0.014	0.022	0.029	0.037

Table 12. Comparison of Sensor and RTF Maintenance Policies on Availability.

When failures are not observable Table 13 displays 1 minus the ratio of the component's maximum operational availability with inspection only to that for a policy with both inspection and a sensor. Results from Tables 4 through 7 in Chapter V as well as Appendix A suggest that when failures are not observable five parameters influence the ability of a sensor to increase operational availability. These parameters are the three parameters from the previous case when failures are observable as well as two other ratios. Parameter four is the ratio of mean time required to conduct an inspection to the mean time required for repair due to failure. The fifth parameter is the ratio of mean time to conduct an inspection to mean time required to repair due to a warning. For fixed Weibull shape parameter and median, smaller ratios result in increases in operational availability.

In Table 13 nearly all entries have positive values reflecting an increased operational availability with the use of the sensor. The exception is for cases 1-3 when the shape parameter is 0.25. The inspection only policy has a higher operational availability when the shape parameter of Weibull distribution of the ratio of warning to failure times is small. This is due to the greater warning time variability resulting from the small shape parameter. Additionally, for these cases the mean time to conduct an inspection is closest to the mean time to repair due to failure. The rest of the Table entries show that the addition of a sensor improves the operational availability when a component's failures are unobserved; this is due to the elimination of the down time from when the component fails until the next inspection.

The ratio of the maximum long run average costs for inspection only to that of inspection and sensor are displayed in Table 25, Appendix A. The results show that cost savings can be obtained when the Weibull shape parameter of the distribution of the ratio of warning to failure time is one or greater regardless of the mean inspection, warning, and failure costs considered in this thesis.

Comparison of Inspection and Sensor & Inspection Maintenance Policies on Availability (1-(Inspect / Sense & Inspect))

Case Numbers for Time and Cost Parameters

Shape	Median	1	2	3	4	5	6	7	8	9	10	11	12
0.25	0.5	-0.003	-0.001	0.000	0.001	0.003	0.004	0.003	0.005	0.006	0.007	0.008	0.010
	0.8	-0.002	-0.001	0.000	0.002	0.003	0.004	0.004	0.005	0.006	0.007	0.008	0.009
	0.9	-0.002	-0.001	0.000	0.002	0.003	0.004	0.004	0.005	0.006	0.007	0.008	0.009
0.5	0.5	0.009	0.012	0.014	0.017	0.019	0.021	0.020	0.023	0.025	0.027	0.029	0.031
	0.8	0.008	0.010	0.012	0.014	0.016	0.018	0.017	0.019	0.020	0.022	0.024	0.026
	0.9	0.008	0.010	0.011	0.013	0.015	0.017	0.016	0.018	0.019	0.021	0.023	0.024
1	0.5	0.031	0.036	0.040	0.044	0.048	0.052	0.051	0.055	0.059	0.063	0.067	0.070
	0.8	0.021	0.024	0.027	0.030	0.033	0.036	0.034	0.037	0.040	0.042	0.045	0.047
	0.9	0.019	0.022	0.025	0.027	0.030	0.032	0.031	0.033	0.036	0.038	0.040	0.043
2	0.5	0.074	0.081	0.087	0.094	0.100	0.107	0.110	0.116	0.122	0.128	0.134	0.140
	0.8	0.038	0.043	0.047	0.051	0.055	0.059	0.057	0.061	0.065	0.068	0.072	0.076
	0.9	0.031	0.035	0.038	0.042	0.045	0.049	0.047	0.050	0.053	0.056	0.059	0.062
5	0.5	0.109	0.117	0.124	0.131	0.138	0.146	0.156	0.163	0.169	0.176	0.182	0.189
	0.8	0.077	0.083	0.090	0.096	0.102	0.108	0.109	0.115	0.121	0.127	0.132	0.138
	0.9	0.051	0.056	0.061	0.066	0.071	0.075	0.073	0.078	0.082	0.087	0.091	0.095
10	0.5	0.109	0.116	0.124	0.131	0.138	0.145	0.156	0.163	0.169	0.176	0.182	0.189
	0.8	0.119	0.127	0.134	0.141	0.148	0.155	0.165	0.172	0.178	0.185	0.191	0.197
	0.9	0.079	0.085	0.092	0.098	0.104	0.110	0.110	0.116	0.122	0.128	0.134	0.139

Shape	Median	13	14	15	16	17	18	19	20	21	22	23	24
0.25	0.5	0.008	0.009	0.011	0.012	0.013	0.014	0.012	0.013	0.014	0.015	0.017	0.018
	0.8	0.008	0.009	0.010	0.011	0.012	0.013	0.011	0.012	0.013	0.014	0.015	0.016
	0.9	0.008	0.009	0.010	0.011	0.012	0.013	0.011	0.012	0.013	0.014	0.015	0.016
0.5	0.5	0.029	0.031	0.033	0.035	0.037	0.039	0.036	0.038	0.040	0.042	0.043	0.045
	0.8	0.024	0.025	0.027	0.028	0.030	0.032	0.029	0.031	0.032	0.034	0.035	0.037
	0.9	0.022	0.024	0.025	0.027	0.029	0.030	0.028	0.029	0.030	0.032	0.033	0.035
1	0.5	0.066	0.070	0.073	0.077	0.080	0.084	0.079	0.082	0.085	0.089	0.092	0.095
	0.8	0.044	0.047	0.049	0.052	0.054	0.056	0.052	0.055	0.057	0.059	0.062	0.064
	0.9	0.040	0.042	0.044	0.046	0.049	0.051	0.047	0.049	0.051	0.053	0.055	0.057
2	0.5	0.137	0.142	0.148	0.154	0.159	0.165	0.159	0.164	0.169	0.174	0.180	0.185
	0.8	0.071	0.075	0.078	0.082	0.085	0.088	0.083	0.086	0.089	0.093	0.096	0.099
	0.9	0.058	0.061	0.064	0.067	0.070	0.073	0.068	0.070	0.073	0.076	0.079	0.082
5	0.5	0.190	0.196	0.202	0.208	0.214	0.220	0.216	0.222	0.228	0.234	0.239	0.245
	0.8	0.133	0.139	0.144	0.149	0.155	0.160	0.153	0.158	0.163	0.168	0.173	0.178
	0.9	0.090	0.094	0.098	0.102	0.106	0.110	0.103	0.107	0.111	0.115	0.119	0.122
10	0.5	0.190	0.196	0.202	0.208	0.214	0.220	0.217	0.222	0.228	0.234	0.239	0.245
	0.8	0.198	0.204	0.210	0.216	0.222	0.228	0.224	0.230	0.235	0.241	0.246	0.252
	0.9	0.134	0.139	0.145	0.150	0.155	0.161	0.153	0.158	0.163	0.168	0.173	0.178

Table 13. Comparison of Inspection and Inspection and Sensor Maintenance Policies on Availability.

B. FUTURE ANALYSIS

1. Model Refinement and Sensor Cost

Refinement of the models to include increasing hazard failure time distributions with resulting age replacement policies remain to be investigated. Further models that include representation of component condition are of interest. Models including more than one component monitored by a sensor are also of interest. Future models should include the cost of a sensor and the maintenance cost of the sensor for systems having failures that are both observable and not observable.

2. Apply to specific systems

Study of component wear and failure data with modes of failures and the performance of sensors to provide warning would produce more insight on the value of implementing Condition Based Maintenance policies for a given system. The author acknowledges that the difficult part of CBM+ is determining what constitutes a warning signal through the noise of the data collected.

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APPENDIX A: ANALYTIC DATA OUTPUT

Tables 14–21 display the output from the VBA code in Appendix G. The parameters of the Weibull distribution are along the left side and the cases being considered are across the top, the numbers listed below the cases from Table 3 are mean times or costs for Inspection, Warning, and Failure (I,W,F). For each set of parameters, results for the four maintenance policies are listed. Results in the table are operational availability, A_O; optimal inter-inspection time T^* (where appropriate); and long run average cost (LRAC) for the policy. Where "INF" is listed as the optimal inspection time it represents an infinite inter-inspection interval and occurs when the probability that a warning occurs before a failure is one. The repeating vertical order of the policies is failure is observable (Run to Failure, and Sensor); failure is not observable (Inspection only and Inspection & Sensor).

				CASE 1			CASE 2			CASE 3			CASE 4			CASE 5			CASE 6	
				I=1			I=1			I=1			I=1			I=1			I=1	
				W=2			W=2			W=2			W=2			W=2			W=2	
				F=2			F=3			F=4			F=5			F=6			F=7	
Shape	**Median**	**Maintenance**	**Ao**	**T***	**LRAC**	**Ao**	**T***	**LRAC**	**Ao**	**T***	**LRAC**	**Ao**	**T***	**LRAC**	**Ao**	**T***	**LRAC**	**Ao**	**T***	**LRAC**
0.25	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.963		0.037	0.955		0.045	0.948		0.052	0.940		0.060	0.933		0.067	0.926		0.074
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.854	15	0.091	0.847	15	0.098	0.842	15	0.104	0.836	15	0.110	0.830	15	0.117	0.824	15	0.123
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.966		0.034	0.958		0.042	0.950		0.050	0.942		0.058	0.935		0.065	0.927		0.073
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.854	15	0.089	0.848	15	0.096	0.842	15	0.103	0.836	15	0.109	0.830	15	0.115	0.824	15	0.122
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.966		0.034	0.958		0.042	0.950		0.050	0.943		0.057	0.935		0.065	0.928		0.072
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.854	15	0.089	0.848	15	0.096	0.842	15	0.102	0.836	15	0.109	0.830	15	0.115	0.824	15	0.121
0.5	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.964		0.036	0.957		0.043	0.951		0.049	0.945		0.055	0.939		0.061	0.932		0.068
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.864	16	0.086	0.859	16	0.092	0.854	16	0.097	0.849	16	0.102	0.844	16	0.108	0.839	16	0.113
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.968		0.032	0.961		0.039	0.954		0.046	0.947		0.053	0.940		0.060	0.934		0.066
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.863	16	0.083	0.857	16	0.089	0.852	16	0.095	0.846	16	0.101	0.841	16	0.107	0.836	16	0.112
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.969		0.031	0.962		0.038	0.955		0.045	0.948		0.052	0.941		0.059	0.934		0.066
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.863	16	0.083	0.857	16	0.089	0.851	16	0.095	0.846	16	0.101	0.840	16	0.106	0.835	16	0.112
1	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.964		0.036	0.960		0.040	0.956		0.044	0.952		0.048	0.947		0.053	0.943		0.057
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.884	20	0.074	0.880	20	0.078	0.877	20	0.081	0.873	20	0.085	0.870	20	0.089	0.866	20	0.093
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.971		0.029	0.965		0.035	0.959		0.041	0.954		0.046	0.948		0.052	0.942		0.058
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.875	17	0.077	0.870	17	0.082	0.865	17	0.087	0.861	17	0.092	0.856	17	0.097	0.851	17	0.102
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.972		0.028	0.966		0.034	0.960		0.040	0.954		0.046	0.948		0.052	0.942		0.058
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.873	17	0.076	0.868	17	0.082	0.863	17	0.087	0.858	17	0.092	0.853	17	0.097	0.848	17	0.102

Table 14. Data Output for Cases 1-6 part 1

				CASE 1			CASE 2			CASE 3			CASE 4			CASE 5			CASE 6	
				I=1			I=1			I=1			I=1			I=1			I=1	
				W=2			W=2			W=2			W=2			W=2			W=2	
				F=2			F=3			F=4			F=5			F=6			F=7	
Shape	Median	Maintenance	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC
2	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.963		0.037	0.962		0.038	0.961		0.039	0.960		0.040	0.959		0.041	0.958		0.042
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.924	38	0.053	0.923	38	0.054	0.922	38	0.056	0.921	38	0.057	0.920	38	0.058	0.919	38	0.059
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.973		0.027	0.969		0.031	0.965		0.035	0.960		0.040	0.956		0.044	0.952		0.048
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.890	20	0.067	0.887	20	0.071	0.883	20	0.075	0.879	20	0.079	0.876	20	0.082	0.872	20	0.086
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.975		0.025	0.970		0.030	0.965		0.035	0.959		0.041	0.954		0.046	0.949		0.051
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.884	18	0.071	0.880	18	0.076	0.875	18	0.080	0.871	18	0.085	0.867	18	0.089	0.863	18	0.093
5	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.975		0.025	0.974		0.026	0.972		0.028	0.971		0.029	0.969		0.031	0.968		0.032
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.927	34	0.047	0.926	34	0.048	0.925	34	0.050	0.923	34	0.051	0.922	34	0.052	0.921	34	0.054
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.977		0.023	0.974		0.026	0.970		0.030	0.967		0.033	0.963		0.037	0.960		0.040
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.902	23	0.059	0.899	23	0.062	0.896	23	0.065	0.894	23	0.068	0.891	23	0.071	0.888	23	0.074
10	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.975		0.025	0.975		0.025	0.975		0.025	0.975		0.025	0.975		0.025	0.975		0.025
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.972	197	0.026	0.972	197	0.026	0.972	197	0.026	0.972	197	0.026	0.972	197	0.026	0.972	197	0.026
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.978		0.022	0.976		0.024	0.975		0.025	0.973		0.027	0.972		0.028	0.970		0.030
		Inspection	0.856	14	0.083	0.849	14	0.090	0.842	14	0.098	0.835	14	0.106	0.828	14	0.113	0.821	14	0.120
		I & S	0.929	34	0.045	0.928	34	0.046	0.927	34	0.048	0.925	34	0.049	0.924	34	0.050	0.923	34	0.052

Table 15. Data Output for Cases 1-6 part 2

				CASE 7			CASE 8			CASE 9			CASE 10			CASE 11			CASE 12	
				I=2			I=2			I=2			I=2			I=2			I=2	
				W=2			W=2			W=2			W=2			W=2			W=2	
				F=2			F=3			F=4			F=5			F=6			F=7	
Shape	Median	Maintenance	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC
0.25	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.963		0.037	0.955		0.045	0.948		0.052	0.940		0.060	0.933		0.067	0.926		0.074
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.813	21	0.112	0.808	21	0.118	0.802	21	0.124	0.797	21	0.130	0.792	21	0.136	0.787	21	0.142
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.966		0.034	0.958		0.042	0.950		0.050	0.942		0.058	0.935		0.065	0.927		0.073
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.814	21	0.111	0.808	21	0.117	0.802	21	0.123	0.797	21	0.129	0.791	21	0.135	0.786	21	0.141
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.966		0.034	0.958		0.042	0.950		0.050	0.943		0.057	0.935		0.065	0.928		0.072
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.814	21	0.110	0.808	21	0.117	0.802	21	0.123	0.797	21	0.129	0.791	21	0.135	0.786	21	0.140
0.5	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.964		0.036	0.957		0.043	0.951		0.049	0.945		0.055	0.939		0.061	0.932		0.068
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.828	23	0.103	0.823	23	0.108	0.818	23	0.113	0.813	23	0.118	0.809	23	0.123	0.804	23	0.128
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.968		0.032	0.961		0.039	0.954		0.046	0.947		0.053	0.940		0.060	0.934		0.066
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.825	22	0.104	0.819	22	0.110	0.814	22	0.115	0.809	22	0.121	0.804	22	0.126	0.800	22	0.131
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.969		0.031	0.962		0.038	0.955		0.045	0.948		0.052	0.941		0.059	0.934		0.066
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.824	22	0.104	0.819	22	0.109	0.814	22	0.115	0.808	22	0.120	0.803	22	0.126	0.798	22	0.131
1	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.964		0.036	0.960		0.040	0.956		0.044	0.952		0.048	0.947		0.053	0.943		0.057
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.854	28	0.088	0.851	28	0.091	0.848	28	0.095	0.844	28	0.098	0.841	28	0.102	0.838	28	0.105
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.971		0.029	0.965		0.035	0.959		0.041	0.954		0.046	0.948		0.052	0.942		0.058
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.839	24	0.095	0.835	24	0.099	0.831	24	0.104	0.826	24	0.109	0.822	24	0.113	0.818	24	0.118
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.972		0.028	0.966		0.034	0.960		0.040	0.954		0.046	0.948		0.052	0.942		0.058
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.836	23	0.097	0.832	23	0.102	0.827	23	0.107	0.823	23	0.112	0.818	23	0.117	0.814	23	0.122

Table 16. Data Output for Cases 7-12 part 1

			CASE 7 I=2 W=2 F=2			CASE 8 I=2 W=2 F=3			CASE 9 I=2 W=2 F=4			CASE 10 I=2 W=2 F=5			CASE 11 I=2 W=2 F=6			CASE 12 I=2 W=2 F=7		
Shape	Median	Maintenance	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC
2	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.963		0.037	0.962		0.038	0.961		0.039	0.960		0.040	0.959		0.041	0.958		0.042
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.911	51	0.059	0.910	51	0.060	0.909	51	0.061	0.908	51	0.062	0.907	51	0.063	0.906	51	0.064
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.973		0.027	0.969		0.031	0.965		0.035	0.960		0.040	0.956		0.044	0.952		0.048
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.860	28	0.082	0.856	28	0.086	0.853	28	0.089	0.849	28	0.093	0.846	28	0.096	0.843	28	0.100
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.975		0.025	0.970		0.030	0.965		0.035	0.959		0.041	0.954		0.046	0.949		0.051
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.850	26	0.086	0.846	26	0.091	0.842	26	0.095	0.839	26	0.099	0.835	26	0.103	0.831	26	0.107
5	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.975		0.025	0.974		0.026	0.972		0.028	0.971		0.029	0.969		0.031	0.968		0.032
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.910	47	0.054	0.909	47	0.056	0.907	47	0.057	0.906	47	0.058	0.905	47	0.060	0.904	47	0.061
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.977		0.023	0.974		0.026	0.970		0.030	0.967		0.033	0.963		0.037	0.960		0.040
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.875	31	0.074	0.872	31	0.077	0.869	31	0.080	0.866	31	0.082	0.864	31	0.085	0.861	31	0.088
10	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.975		0.025	0.975		0.025	0.975		0.025	0.975		0.025	0.975		0.025	0.975		0.025
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.971	242	0.027	0.971	242	0.027	0.971	242	0.027	0.971	242	0.027	0.971	242	0.027	0.971	242	0.027
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.978		0.022	0.976		0.024	0.975		0.025	0.973		0.027	0.972		0.028	0.970		0.030
		Inspection	0.811	19	0.110	0.804	19	0.117	0.798	19	0.124	0.791	19	0.131	0.785	19	0.138	0.779	19	0.145
		I & S	0.911	47	0.053	0.910	47	0.054	0.909	47	0.056	0.907	47	0.057	0.906	47	0.058	0.905	47	0.060

Table 17. Data Output for Cases 7-12 part 2

			CASE 13 I=3 W=2 F=2			CASE 14 I=3 W=2 F=3			CASE 15 I=3 W=2 F=4			CASE 16 I=3 W=2 F=5			CASE 17 I=3 W=2 F=6			CASE 18 I=3 W=2 F=7		
Shape	Median	Maintenance	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC
0.25	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.963		0.037	0.955		0.045	0.948		0.052	0.940		0.060	0.933		0.067	0.926		0.074
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.784	25	0.130	0.779	25	0.135	0.774	25	0.141	0.769	25	0.147	0.764	25	0.152	0.759	25	0.158
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.966		0.034	0.958		0.042	0.950		0.050	0.942		0.058	0.935		0.065	0.927		0.073
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.784	25	0.128	0.779	25	0.134	0.774	25	0.140	0.769	25	0.146	0.764	25	0.151	0.759	25	0.157
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.966		0.034	0.958		0.042	0.950		0.050	0.943		0.057	0.935		0.065	0.928		0.072
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.784	25	0.128	0.779	25	0.134	0.774	25	0.140	0.768	25	0.145	0.763	25	0.151	0.758	25	0.157
0.5	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.964		0.036	0.957		0.043	0.951		0.049	0.945		0.055	0.939		0.061	0.932		0.068
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.801	28	0.117	0.797	28	0.122	0.792	28	0.126	0.788	28	0.131	0.783	28	0.136	0.779	28	0.141
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.968		0.032	0.961		0.039	0.954		0.046	0.947		0.053	0.940		0.060	0.934		0.066
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.797	27	0.118	0.792	27	0.123	0.787	27	0.128	0.783	27	0.134	0.778	27	0.139	0.773	27	0.144
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.969		0.031	0.962		0.038	0.955		0.045	0.948		0.052	0.941		0.059	0.934		0.066
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.796	26	0.121	0.791	26	0.126	0.786	26	0.132	0.781	26	0.137	0.777	26	0.142	0.772	26	0.147
1	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.964		0.036	0.960		0.040	0.956		0.044	0.952		0.048	0.947		0.053	0.943		0.057
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.833	33	0.100	0.830	33	0.103	0.827	33	0.107	0.824	33	0.110	0.821	33	0.113	0.817	33	0.117
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.971		0.029	0.965		0.035	0.959		0.041	0.954		0.046	0.948		0.052	0.942		0.058
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.814	29	0.108	0.810	29	0.113	0.806	29	0.117	0.802	29	0.122	0.798	29	0.126	0.794	29	0.131
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.972		0.028	0.966		0.034	0.960		0.040	0.954		0.046	0.948		0.052	0.942		0.058
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.810	28	0.111	0.806	28	0.116	0.802	28	0.121	0.797	28	0.125	0.793	28	0.130	0.789	28	0.134

Table 18. Data Output for Cases 13-18 part 1

				CASE 13			CASE 14			CASE 15			CASE 16			CASE 17			CASE 18	
				I=3			I=3			I=3			I=3			I=3			I=3	
				W=2			W=2			W=2			W=2			W=2			W=2	
				F=2			F=3			F=4			F=5			F=6			F=7	
Shape	**Median**	**Maintenance**	**Ao**	**T***	**LRAC**	**Ao**	**T***	**LRAC**	**Ao**	**T***	**LRAC**	**Ao**	**T***	**LRAC**	**Ao**	**T***	**LRAC**	**Ao**	**T***	**LRAC**
2	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.963		0.037	0.962		0.038	0.961		0.039	0.960		0.040	0.959		0.041	0.958		0.042
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.901	61	0.063	0.900	61	0.064	0.899	61	0.065	0.898	61	0.066	0.897	61	0.067	0.896	61	0.068
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.973		0.027	0.969		0.031	0.965		0.035	0.960		0.040	0.956		0.044	0.952		0.048
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.837	34	0.093	0.834	34	0.096	0.831	34	0.100	0.828	34	0.103	0.825	34	0.107	0.821	34	0.110
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.975		0.025	0.970		0.030	0.965		0.035	0.959		0.041	0.954		0.046	0.949		0.051
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.826	31	0.100	0.822	31	0.104	0.819	31	0.108	0.815	31	0.112	0.811	31	0.116	0.808	31	0.120
5	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.975		0.025	0.974		0.026	0.972		0.028	0.971		0.029	0.969		0.031	0.968		0.032
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.897	56	0.060	0.896	56	0.061	0.895	56	0.063	0.894	56	0.064	0.893	56	0.065	0.891	56	0.067
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.977		0.023	0.974		0.026	0.970		0.030	0.967		0.033	0.963		0.037	0.960		0.040
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.855	38	0.083	0.852	38	0.086	0.849	38	0.089	0.847	38	0.091	0.844	38	0.094	0.842	38	0.097
10	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.975		0.025	0.975		0.025	0.975		0.025	0.975		0.025	0.975		0.025	0.975		0.025
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.970	269	0.028	0.970	269	0.028	0.970	269	0.028	0.970	269	0.028	0.970	269	0.028	0.970	269	0.028
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.978		0.022	0.976		0.024	0.975		0.025	0.973		0.027	0.972		0.028	0.970		0.030
		Inspection	0.778	24	0.125	0.772	24	0.132	0.766	24	0.138	0.760	24	0.145	0.754	24	0.151	0.749	24	0.158
		I & S	0.898	56	0.059	0.897	56	0.060	0.896	56	0.062	0.895	56	0.063	0.893	56	0.064	0.892	56	0.066

Table 19. Data Output for Cases 13-18 part 2

				CASE 19			CASE 20			CASE 21			CASE 22			CASE 23			CASE 24	
				I=4			I=4			I=4			I=4			I=4			I=4	
				W=2			W=2			W=2			W=2			W=2			W=2	
				F=2			F=3			F=4			F=5			F=6			F=7	
Shape	**Median**	**Maintenance**	**Ao**	**T***	**LRAC**	**Ao**	**T***	**LRAC**	**Ao**	**T***	**LRAC**	**Ao**	**T***	**LRAC**	**Ao**	**T***	**LRAC**	**Ao**	**T***	**LRAC**
0.25	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.963		0.037	0.955		0.045	0.948		0.052	0.940		0.060	0.933		0.067	0.926		0.074
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.761	29	0.142	0.756	29	0.147	0.751	29	0.153	0.747	29	0.158	0.742	29	0.163	0.738	29	0.168
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.966		0.034	0.958		0.042	0.950		0.050	0.942		0.058	0.935		0.065	0.927		0.073
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.760	29	0.141	0.755	29	0.146	0.751	29	0.152	0.746	29	0.157	0.741	29	0.162	0.736	29	0.168
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.966		0.034	0.958		0.042	0.950		0.050	0.943		0.057	0.935		0.065	0.928		0.072
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.760	29	0.140	0.755	29	0.146	0.750	29	0.151	0.746	29	0.157	0.741	29	0.162	0.736	29	0.168
0.5	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.964		0.036	0.957		0.043	0.951		0.049	0.945		0.055	0.939		0.061	0.932		0.068
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.780	32	0.128	0.775	32	0.133	0.771	32	0.138	0.767	32	0.142	0.763	32	0.147	0.759	32	0.151
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.968		0.032	0.961		0.039	0.954		0.046	0.947		0.053	0.940		0.060	0.934		0.066
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.774	31	0.130	0.770	31	0.135	0.765	31	0.140	0.761	31	0.145	0.756	31	0.150	0.752	31	0.155
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.969		0.031	0.962		0.038	0.955		0.045	0.948		0.052	0.941		0.059	0.934		0.066
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.773	30	0.133	0.769	30	0.138	0.764	30	0.143	0.759	30	0.148	0.755	30	0.153	0.751	30	0.158
1	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.964		0.036	0.960		0.040	0.956		0.044	0.952		0.048	0.947		0.053	0.943		0.057
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.816	38	0.108	0.813	38	0.111	0.810	38	0.115	0.807	38	0.118	0.804	38	0.121	0.801	38	0.124
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.971		0.029	0.965		0.035	0.959		0.041	0.954		0.046	0.948		0.052	0.942		0.058
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.793	33	0.120	0.789	33	0.124	0.785	33	0.129	0.782	33	0.133	0.778	33	0.137	0.774	33	0.141
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.972		0.028	0.966		0.034	0.960		0.040	0.954		0.046	0.948		0.052	0.942		0.058
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.789	33	0.120	0.785	33	0.125	0.781	33	0.129	0.777	33	0.134	0.773	33	0.138	0.769	33	0.142

Table 20. Data Output for Cases 19-24 part 1

			CASE 19			CASE 20			CASE 21			CASE 22			CASE 23			CASE 24		
			I=4 W=2 F=2			I=4 W=2 F=3			I=4 W=2 F=4			I=4 W=2 F=5			I=4 W=2 F=6			I=4 W=2 F=7		
Shape	Median	Maintenance	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC	Ao	T*	LRAC
2	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.963		0.037	0.962		0.038	0.961		0.039	0.960		0.040	0.959		0.041	0.958		0.042
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.894	68	0.066	0.893	68	0.067	0.892	68	0.068	0.891	68	0.069	0.890	68	0.070	0.889	68	0.071
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.973		0.027	0.969		0.031	0.965		0.035	0.960		0.040	0.956		0.044	0.952		0.048
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.819	39	0.102	0.816	39	0.105	0.813	39	0.109	0.810	39	0.112	0.807	39	0.115	0.804	39	0.119
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.975		0.025	0.970		0.030	0.965		0.035	0.959		0.041	0.954		0.046	0.949		0.051
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.806	36	0.109	0.803	36	0.113	0.799	36	0.117	0.796	36	0.121	0.792	36	0.125	0.789	36	0.129
5	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.975		0.025	0.974		0.026	0.972		0.028	0.971		0.029	0.969		0.031	0.968		0.032
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.887	64	0.064	0.886	64	0.065	0.885	64	0.067	0.884	64	0.068	0.883	64	0.069	0.881	64	0.071
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.977		0.023	0.974		0.026	0.970		0.030	0.967		0.033	0.963		0.037	0.960		0.040
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.838	43	0.092	0.836	43	0.095	0.833	43	0.097	0.831	43	0.100	0.828	43	0.103	0.826	43	0.105
10	0.5	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039	0.961		0.039
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039	0.961	INF	0.039
	0.8	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.975		0.025	0.975		0.025	0.975		0.025	0.975		0.025	0.975		0.025	0.975		0.025
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.970	290	0.026	0.970	290	0.026	0.970	290	0.026	0.970	290	0.026	0.970	290	0.026	0.970	290	0.026
	0.9	RTF	0.980		0.020	0.971		0.029	0.962		0.038	0.952		0.048	0.943		0.057	0.935		0.065
		Sensor	0.978		0.022	0.976		0.024	0.975		0.025	0.973		0.027	0.972		0.028	0.970		0.030
		Inspection	0.752	27	0.142	0.746	27	0.149	0.741	27	0.155	0.735	27	0.161	0.730	27	0.167	0.725	27	0.173
		I & S	0.888	64	0.063	0.887	64	0.065	0.885	64	0.066	0.884	64	0.067	0.883	64	0.069	0.882	64	0.070

Table 21. Data Output for Cases 19-24 part 2

Tables 22–25 are comparison tables of the different maintenance policies. The tables compare availability and long run average cost for policies in which failures are observable and the policies when the failures are not observable. The parameters of the Weibull distribution are along the left side and the cases being considered, from Table 3, are across the top. When failures are not observed, as in Tables 22 & 24, the inspection times/costs do not affect the availability or long run average cost; hence the multiple cases are listed at the top of each column.

Table 22 displays 1 minus the ratio of the operational availability without a sensor to that with a sensor for a component whose failures are observable. Negative values result when the operational availability of the component with a sensor is less than that without a sensor due to premature warnings.

Table 23 displays 1 minus the ratio of the maximum operational availability with only inspections to that with a sensor and inspections for a component

whose failures are not observable. Negative values result when the maximum operational availability of the component with a sensor and inspections is less than that with inspections only due to sensor warnings being either too early or not occurring at all.

Table 24 displays the ratio of the long run average cost without a sensor to that with a sensor for a component whose failures are observable; a ratio less than one indicates that the long run average cost with a sensor is greater than that without a sensor.

Table 25 displays the ratio of the minimum long run average cost without a sensor to that with a sensor for a component whose failures are not observable; a ratio less than one indicates that the long run average cost with a sensor is greater than that without a sensor.

Comparison of Sensor and Run to Failure Maintenance Policies on Availability (1-(RTF / Sensor))

		Case Numbers for Time and Cost Parameters					
Shape	Median	1,7,13,19	2,8,14,20	3,9,15,21	4,10,16,22	5,11,17,23	6,12,18,24
0.25	0.5	-0.018	-0.016	-0.015	-0.013	-0.011	-0.010
	0.8	-0.015	-0.014	-0.012	-0.011	-0.009	-0.008
	0.9	-0.015	-0.013	-0.012	-0.010	-0.009	-0.008
0.5	0.5	-0.017	-0.014	-0.011	-0.008	-0.005	-0.002
	0.8	-0.013	-0.010	-0.008	-0.005	-0.003	-0.001
	0.9	-0.012	-0.009	-0.007	-0.005	-0.003	-0.001
1	0.5	-0.017	-0.011	-0.006	-0.001	0.004	0.009
	0.8	-0.010	-0.006	-0.002	0.001	0.005	0.008
	0.9	-0.009	-0.005	-0.002	0.001	0.004	0.008
2	0.5	-0.018	-0.009	-0.001	0.008	0.016	0.024
	0.8	-0.007	-0.002	0.003	0.008	0.013	0.018
	0.9	-0.006	-0.001	0.003	0.007	0.012	0.016
5	0.5	-0.020	-0.010	0.000	0.009	0.018	0.028
	0.8	-0.005	0.003	0.011	0.019	0.027	0.034
	0.9	-0.003	0.003	0.009	0.015	0.021	0.027
10	0.5	-0.020	-0.010	-0.001	0.009	0.018	0.028
	0.8	-0.005	0.005	0.014	0.023	0.033	0.042
	0.9	-0.003	0.006	0.014	0.022	0.029	0.037

Table 22. Comparison of Sensor and Run to Failure Maintenance Policies on Availability

Comparison of Inspection and Sensor & Inspection Maintenance Policies on Availability (1-(Inspect / Sense & Inspect))

Case Numbers for Time and Cost Parameters

Shape	Median	1	2	3	4	5	6	7	8	9	10	11	12
0.25	0.5	-0.003	-0.001	0.000	0.001	0.003	0.004	0.003	0.005	0.006	0.007	0.008	0.010
	0.8	-0.002	-0.001	0.000	0.002	0.003	0.004	0.004	0.005	0.006	0.007	0.008	0.009
	0.9	-0.002	-0.001	0.000	0.002	0.003	0.004	0.004	0.005	0.006	0.007	0.008	0.009
0.5	0.5	0.009	0.012	0.014	0.017	0.019	0.021	0.020	0.023	0.025	0.027	0.029	0.031
	0.8	0.008	0.010	0.012	0.014	0.016	0.018	0.017	0.019	0.020	0.022	0.024	0.026
	0.9	0.008	0.010	0.011	0.013	0.015	0.017	0.016	0.018	0.019	0.021	0.023	0.024
1	0.5	0.031	0.036	0.040	0.044	0.048	0.052	0.051	0.055	0.059	0.063	0.067	0.070
	0.8	0.021	0.024	0.027	0.030	0.033	0.036	0.034	0.037	0.040	0.042	0.045	0.047
	0.9	0.019	0.022	0.025	0.027	0.030	0.032	0.031	0.033	0.036	0.038	0.040	0.043
2	0.5	0.074	0.081	0.087	0.094	0.100	0.107	0.110	0.116	0.122	0.128	0.134	0.140
	0.8	0.038	0.043	0.047	0.051	0.055	0.059	0.057	0.061	0.065	0.068	0.072	0.076
	0.9	0.031	0.035	0.038	0.042	0.045	0.049	0.047	0.050	0.053	0.056	0.059	0.062
5	0.5	0.109	0.117	0.124	0.131	0.138	0.146	0.156	0.163	0.169	0.176	0.182	0.189
	0.8	0.077	0.083	0.090	0.096	0.102	0.108	0.109	0.115	0.121	0.127	0.132	0.138
	0.9	0.051	0.056	0.061	0.066	0.071	0.075	0.073	0.078	0.082	0.087	0.091	0.095
10	0.5	0.109	0.116	0.124	0.131	0.138	0.145	0.156	0.163	0.169	0.176	0.182	0.189
	0.8	0.119	0.127	0.134	0.141	0.148	0.155	0.165	0.172	0.178	0.185	0.191	0.197
	0.9	0.079	0.085	0.092	0.098	0.104	0.110	0.110	0.116	0.122	0.128	0.134	0.139

Shape	Median	13	14	15	16	17	18	19	20	21	22	23	24
0.25	0.5	0.008	0.009	0.011	0.012	0.013	0.014	0.012	0.013	0.014	0.015	0.017	0.018
	0.8	0.008	0.009	0.010	0.011	0.012	0.013	0.011	0.012	0.013	0.014	0.015	0.016
	0.9	0.008	0.009	0.010	0.011	0.012	0.013	0.011	0.012	0.013	0.014	0.015	0.016
0.5	0.5	0.029	0.031	0.033	0.035	0.037	0.039	0.036	0.038	0.040	0.042	0.043	0.045
	0.8	0.024	0.025	0.027	0.028	0.030	0.032	0.029	0.031	0.032	0.034	0.035	0.037
	0.9	0.022	0.024	0.025	0.027	0.029	0.030	0.028	0.029	0.030	0.032	0.033	0.035
1	0.5	0.066	0.070	0.073	0.077	0.080	0.084	0.079	0.082	0.085	0.089	0.092	0.095
	0.8	0.044	0.047	0.049	0.052	0.054	0.056	0.052	0.055	0.057	0.059	0.062	0.064
	0.9	0.040	0.042	0.044	0.046	0.049	0.051	0.047	0.049	0.051	0.053	0.055	0.057
2	0.5	0.137	0.142	0.148	0.154	0.159	0.165	0.159	0.164	0.169	0.174	0.180	0.185
	0.8	0.071	0.075	0.078	0.082	0.085	0.088	0.083	0.086	0.089	0.093	0.096	0.099
	0.9	0.058	0.061	0.064	0.067	0.070	0.073	0.068	0.070	0.073	0.076	0.079	0.082
5	0.5	0.190	0.196	0.202	0.208	0.214	0.220	0.216	0.222	0.228	0.234	0.239	0.245
	0.8	0.133	0.139	0.144	0.149	0.155	0.160	0.153	0.158	0.163	0.168	0.173	0.178
	0.9	0.090	0.094	0.098	0.102	0.106	0.110	0.103	0.107	0.111	0.115	0.119	0.122
10	0.5	0.190	0.196	0.202	0.208	0.214	0.220	0.217	0.222	0.228	0.234	0.239	0.245
	0.8	0.198	0.204	0.210	0.216	0.222	0.228	0.224	0.230	0.235	0.241	0.246	0.252
	0.9	0.134	0.139	0.145	0.150	0.155	0.161	0.153	0.158	0.163	0.168	0.173	0.178

Table 23. Comparison of Inspection and Sensor & Inspection Maintenance Policies on Availability

Comparison of Sensor and Run to Failure Maintenance Policies on Long Run Average Cost (RTF / Sensor)							
		Case Numbers for Time and Cost Parameters					
Shape	Median	1,7,13,19	2,8,14,20	3,9,15,21	4,10,16,22	5,11,17,23	6,12,18,24
0.25	0.5	0.531	0.653	0.736	0.797	0.844	0.880
	0.8	0.569	0.687	0.767	0.823	0.866	0.898
	0.9	0.578	0.696	0.774	0.829	0.871	0.902
0.5	0.5	0.544	0.685	0.786	0.863	0.921	0.968
	0.8	0.615	0.747	0.837	0.902	0.951	0.988
	0.9	0.631	0.761	0.848	0.910	0.956	0.991
1	0.5	0.550	0.729	0.871	0.984	1.077	1.154
	0.8	0.675	0.834	0.944	1.024	1.085	1.132
	0.9	0.703	0.854	0.955	1.028	1.081	1.123
2	0.5	0.532	0.767	0.984	1.185	1.371	1.543
	0.8	0.737	0.940	1.089	1.203	1.291	1.361
	0.9	0.780	0.960	1.085	1.175	1.243	1.295
5	0.5	0.504	0.749	0.988	1.224	1.455	1.681
	0.8	0.786	1.103	1.380	1.623	1.839	2.030
	0.9	0.856	1.105	1.292	1.436	1.549	1.640
10	0.5	0.503	0.747	0.986	1.221	1.452	1.678
	0.8	0.794	1.178	1.554	1.923	2.284	2.638
	0.9	0.885	1.233	1.532	1.792	2.018	2.217

Table 24. Comparison of Sensor and Run to Failure Maintenance Policies on Long Run Average Cost

Comparison of Inspection and Sensor & Inspection Maintenance Policies on Long Run Average Cost (Inspect / Sense & Inspect)													
		Case Numbers for Time and Cost Parameters											
Shape	Median	1	2	3	4	5	6	7	8	9	10	11	12
0.25	0.5	0.905	0.925	0.942	0.956	0.969	0.980	0.979	0.990	0.999	1.007	1.014	1.021
	0.8	0.924	0.941	0.956	0.969	0.980	0.989	0.993	1.002	1.010	1.016	1.022	1.028
	0.9	0.928	0.945	0.959	0.972	0.982	0.991	0.996	1.005	1.012	1.018	1.024	1.029
0.5	0.5	0.960	0.987	1.010	1.031	1.049	1.064	1.066	1.081	1.094	1.106	1.116	1.126
	0.8	0.994	1.014	1.032	1.047	1.060	1.072	1.056	1.068	1.078	1.087	1.095	1.102
	0.9	1.001	1.020	1.036	1.050	1.062	1.073	1.061	1.071	1.081	1.089	1.096	1.102
1	0.5	1.117	1.163	1.204	1.240	1.272	1.300	1.253	1.282	1.309	1.332	1.353	1.372
	0.8	1.070	1.099	1.124	1.146	1.164	1.181	1.161	1.178	1.192	1.205	1.216	1.226
	0.9	1.081	1.105	1.127	1.145	1.160	1.174	1.131	1.146	1.159	1.170	1.180	1.189
2	0.5	1.547	1.660	1.767	1.867	1.962	2.052	1.861	1.949	2.031	2.110	2.184	2.254
	0.8	1.229	1.272	1.310	1.343	1.372	1.397	1.339	1.365	1.389	1.410	1.429	1.446
	0.9	1.159	1.193	1.222	1.248	1.270	1.289	1.271	1.290	1.307	1.322	1.336	1.347
5	0.5	2.102	2.301	2.495	2.687	2.875	3.060	2.768	2.948	3.125	3.300	3.472	3.641
	0.8	1.760	1.872	1.974	2.070	2.157	2.239	2.020	2.100	2.175	2.245	2.310	2.370
	0.9	1.410	1.466	1.515	1.559	1.597	1.631	1.492	1.528	1.561	1.590	1.617	1.641
10	0.5	2.101	2.299	2.494	2.685	2.873	3.058	2.770	2.950	3.128	3.303	3.475	3.644
	0.8	3.208	3.507	3.801	4.090	4.374	4.650	4.110	4.370	4.630	4.886	5.137	5.383
	0.9	1.841	1.954	2.059	2.155	2.244	2.325	2.080	2.160	2.235	2.305	2.370	2.430

Shape	Median	13	14	15	16	17	18	19	20	21	22	23	24
0.25	0.5	0.963	0.972	0.981	0.988	0.995	1.001	1.003	1.009	1.015	1.020	1.025	1.029
	0.8	0.973	0.981	0.988	0.995	1.001	1.006	1.011	1.017	1.021	1.026	1.029	1.033
	0.9	0.975	0.983	0.990	0.996	1.002	1.007	1.013	1.018	1.023	1.027	1.030	1.034
0.5	0.5	1.071	1.084	1.094	1.104	1.113	1.120	1.109	1.118	1.125	1.132	1.139	1.144
	0.8	1.059	1.068	1.077	1.084	1.091	1.096	1.094	1.100	1.106	1.111	1.115	1.119
	0.9	1.033	1.043	1.051	1.059	1.066	1.072	1.069	1.076	1.082	1.087	1.092	1.096
1	0.5	1.251	1.275	1.296	1.316	1.334	1.350	1.317	1.335	1.351	1.366	1.380	1.392
	0.8	1.154	1.167	1.179	1.190	1.200	1.208	1.186	1.195	1.204	1.212	1.219	1.226
	0.9	1.125	1.137	1.148	1.158	1.166	1.174	1.185	1.193	1.200	1.206	1.211	1.216
2	0.5	1.997	2.071	2.142	2.209	2.272	2.333	2.152	2.215	2.275	2.333	2.387	2.439
	0.8	1.344	1.365	1.384	1.401	1.417	1.431	1.396	1.412	1.426	1.439	1.451	1.461
	0.9	1.247	1.263	1.278	1.291	1.302	1.313	1.300	1.312	1.322	1.331	1.340	1.347
5	0.5	3.115	3.284	3.449	3.613	3.774	3.932	3.509	3.667	3.823	3.976	4.127	4.276
	0.8	2.078	2.144	2.205	2.263	2.317	2.368	2.215	2.269	2.320	2.368	2.413	2.455
	0.9	1.507	1.536	1.562	1.586	1.608	1.627	1.547	1.570	1.591	1.610	1.627	1.643
10	0.5	3.122	3.291	3.457	3.620	3.782	3.940	3.521	3.680	3.836	3.990	4.141	4.291
	0.8	4.491	4.731	4.966	5.198	5.426	5.650	5.466	5.713	5.955	6.194	6.430	6.662
	0.9	2.117	2.183	2.245	2.302	2.356	2.407	2.240	2.295	2.346	2.393	2.438	2.480

Table 25. Comparison of Inspection and Sensor & Inspection Maintenance Policies on Long Run Average Cost

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APPENDIX B SELECTED WEIBULL PLOTS

The Weibull random variable mean, median and variance are presented here:

$$Mean\,\varphi = E[\varphi] = \frac{1}{\alpha}\Gamma\left(1+\frac{1}{\beta}\right)$$

$$Median\ \varphi = \frac{1}{\alpha}(\ln(2))^{\frac{1}{\beta}}$$

$$Var\,\varphi = \left(\frac{1}{\alpha}\right)^2\Gamma\left(1+\frac{2}{\beta}\right) - \left(\frac{1}{\alpha}\Gamma\left(1+\frac{1}{\beta}\right)\right)^2$$

Density function plots for the Weibull random variables used in the thesis are displayed in the following tables. For shape parameter less than one the first x value is 0.01, since zero gives an undefined value.

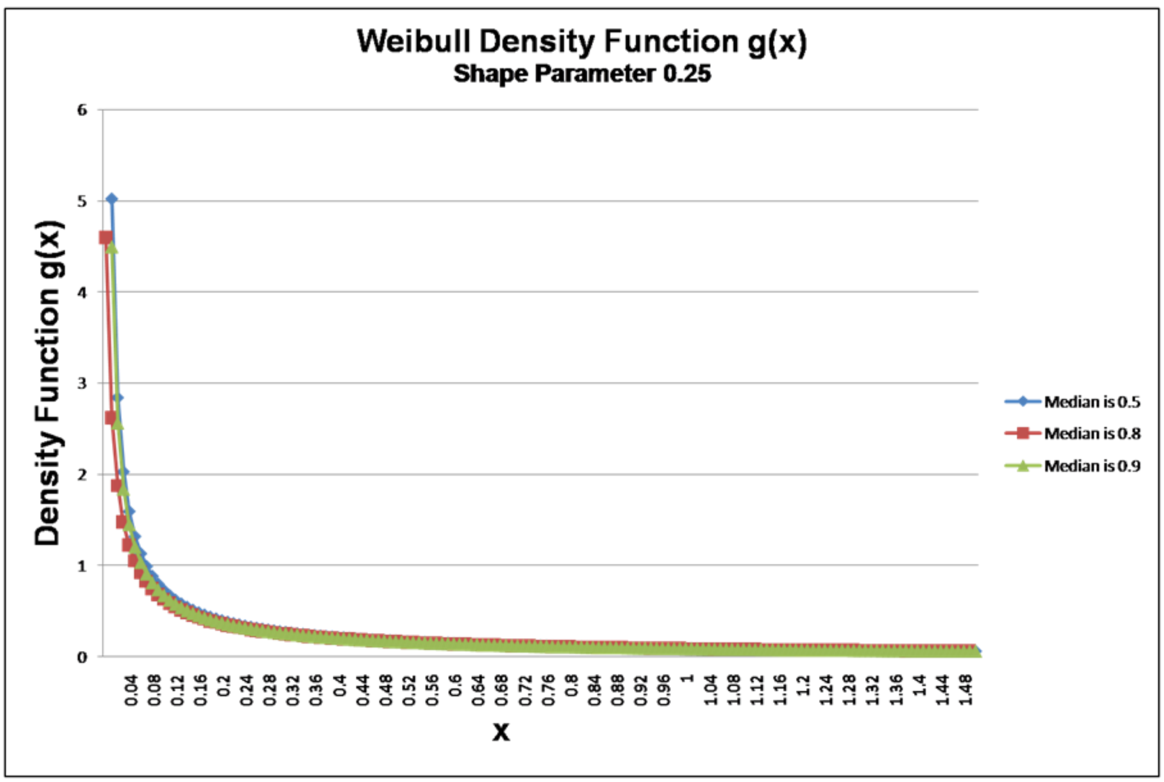

Figure 8. Weibull Random Variable with Shape Parameter 0.25

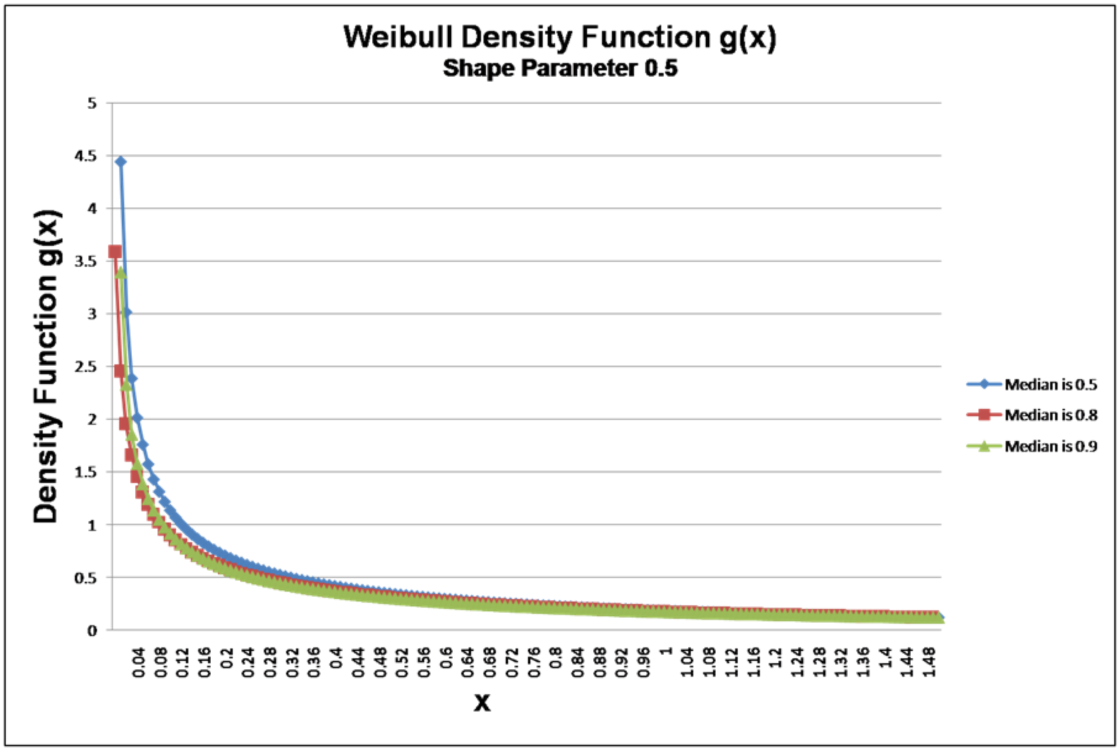

Figure 9. Weibull Random Variable with Shape Parameter 0.5

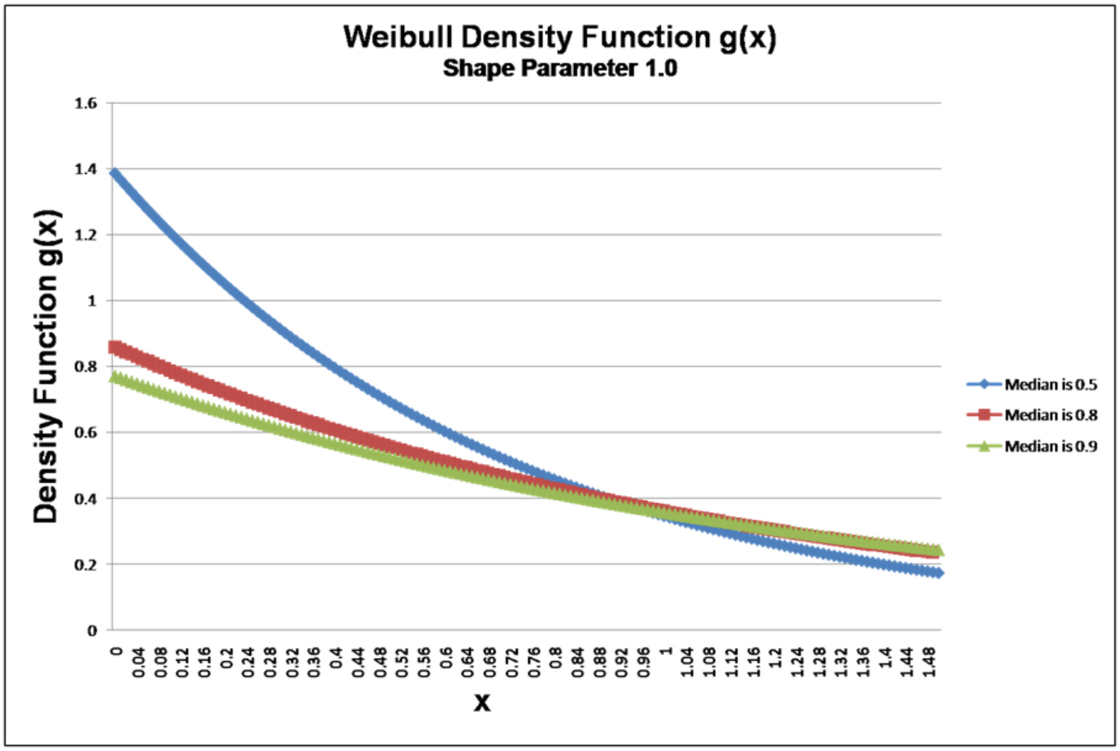

Figure 10. Weibull Random Variable with Shape Parameter 1.0

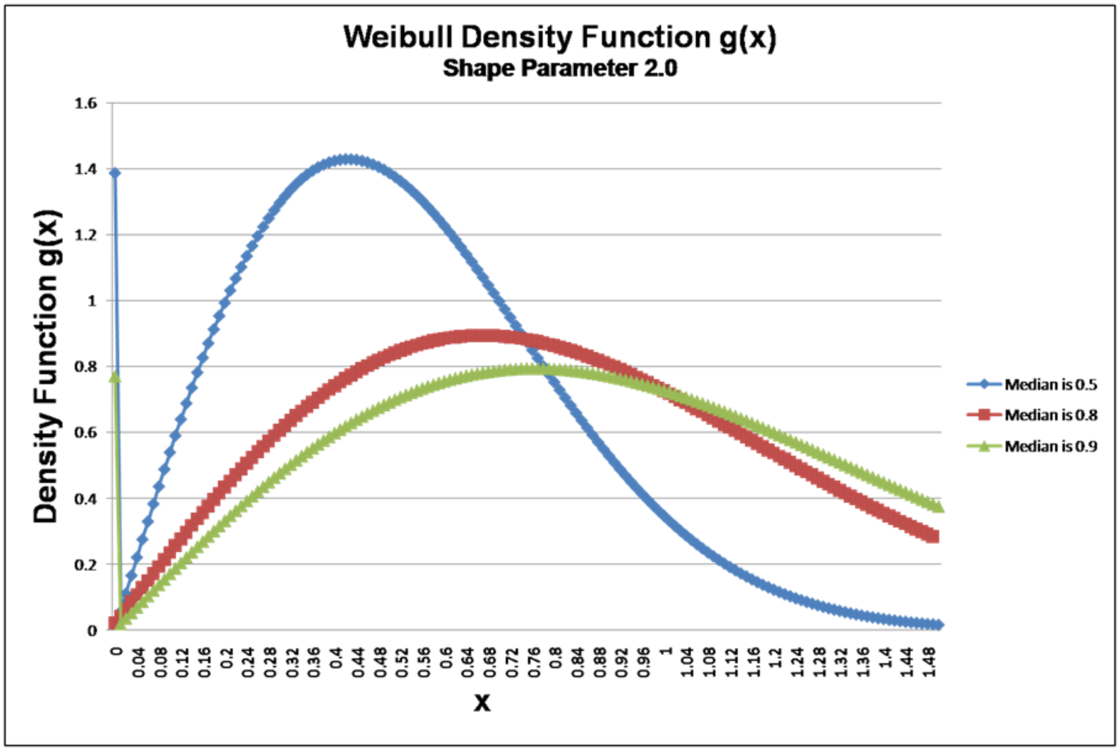

Figure 11. Weibull Random Variable with Shape Parameter 2.0

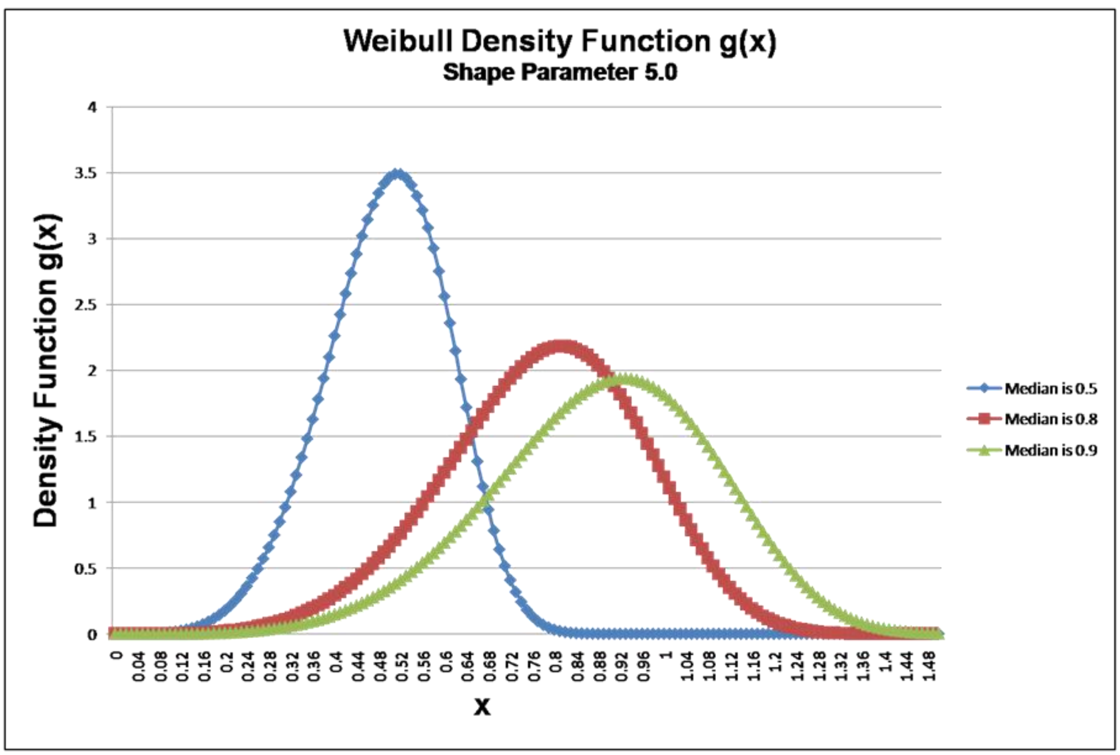

Figure 12. Weibull Random Variable with Shape Parameter 5.0

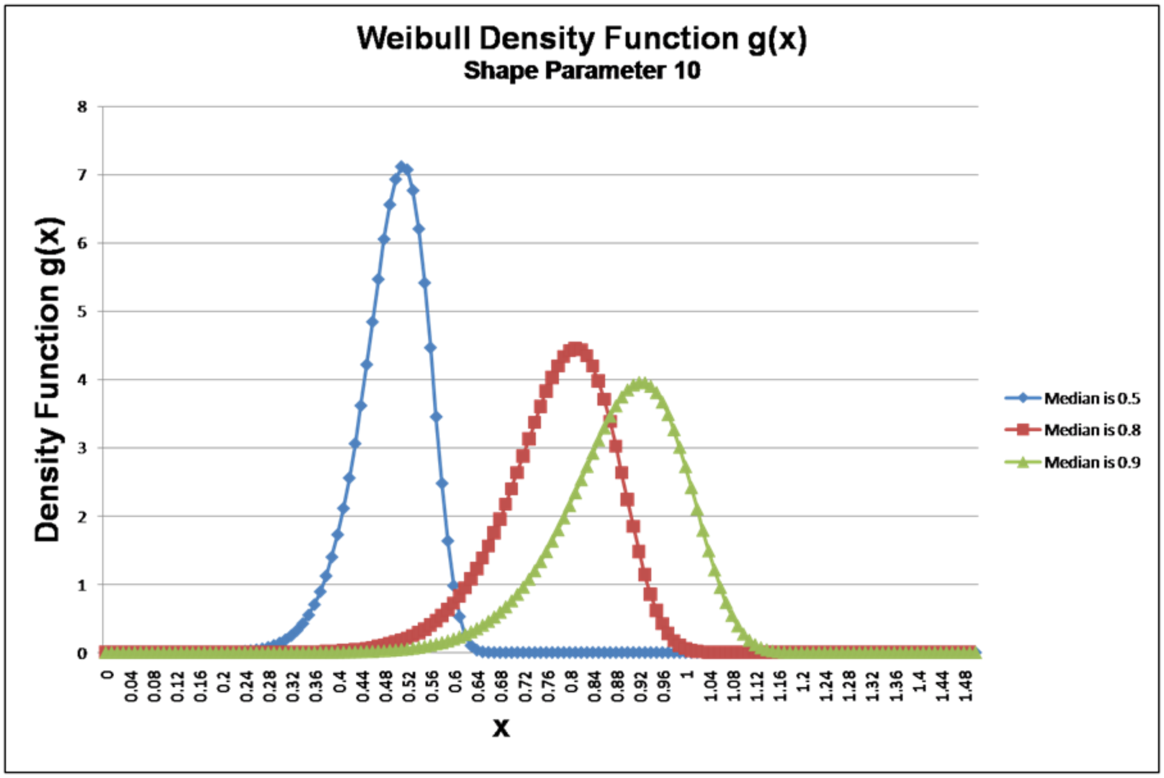

Figure 13. Weibull Random Variable with Shape Parameter 10

The covariance and correlation of T_F, T_W are displayed in Table 26 below for each of the Weibull parameters used in the calculations.

Shape	**Median**	**Covariance** T_F, T_W	**Correlation** T_F, T_W
0.25	0.5	8108012	0.486
	0.8	12975630	0.486
	0.9	14594422	0.486
0.5	0.5	161863	0.753
	0.8	258883	0.753
	0.9	291280	0.753
1.0	0.5	27100	0.911

	0.8	43362	0.911
	0.9	48778	0.911
2	0.5	11539	0.973
	0.8	18462	0.973
	0.9	20769	0.973
5	0.5	6930	0.995
	0.8	11088	0.995
	0.9	12473	0.995
10	0.5	5834	0.999
	0.8	9334	0.999
	0.9	10501	0.999

Table 26. Covariance and Correlation of T_F, T_W

Table 27 displays characteristics for the Weibull random variables used to determine the warning time.

Shape	**Median**	**Mean**	**P $\{T_W < T_F\}$**	**Scale**	**Variance**
0.25	0.5	51.99	0.56	0.4617	1.86×10^5
	0.8	83.18	0.52	0.2885	4.77×10^5
	0.9	93.57	0.51	0.2565	6.04×10^5
0.5	0.5	2.08	0.62	0.9609	21.6605
	0.8	3.33	0.54	0.6006	55.4508
	0.9	3.75	0.52	0.5338	70.1800
1.0	0.5	0.72	0.75	1.3863	0.5203
	0.8	1.15	0.58	0.8664	1.3321

	0.9	1.30	0.54	0.7702	1.6859
2	0.5	0.53	0.94	1.6651	0.0774
	0.8	0.85	0.66	1.0407	0.1981
	0.9	0.96	0.58	0.8516	0.2508
5	0.5	0.49	1.00	1.8586	0.0128
	0.8	0.79	0.88	1.1616	0.0328
	0.9	0.88	0.69	1.0326	0.0415
10	0.5	0.49	1.00	1.9280	0.0035
	0.8	0.79	0.99	1.2050	0.0090
	0.9	0.89	0.86	1.0711	0.0114

Table 27. Sensor Parameter Characteristics

APPENDIX C: SENSOR BASED PREDICTIVE MAINTENANCE MODEL

The failures are observable. When there is a warning or failure, the component is replaced or repaired. The expected up time is the expected value of the minimum of time to warning $\boldsymbol{T}_W$ and the time to failure $\boldsymbol{T}_F$.

$$\boldsymbol{T}_W = \boldsymbol{\varphi}\boldsymbol{T}_F$$

$$E\left[\min(\boldsymbol{T}_W, \boldsymbol{T}_F)\right]$$

$$= E\left[\min(\boldsymbol{\varphi}\boldsymbol{T}_F, \boldsymbol{T}_F)\right]$$

$$= E\left[\boldsymbol{T}_F\right]\left[\int_0^1 sP\left\{\boldsymbol{\varphi} \in ds\right\} + P\left\{\boldsymbol{\varphi} > 1\right\}\right]$$

$$= E\left[\boldsymbol{T}_F\right]\left[\int_0^1 ss^{\beta-1}\alpha^{\beta}\beta e^{-(\alpha s)^{\beta}}\, ds + e^{-\alpha^{\beta}}\right]$$

$$= E\left[\boldsymbol{T}_F\right]\left[\alpha^{\beta}\beta\int_0^1 s^{\beta}e^{-(\alpha s)^{\beta}}\, ds + e^{-(\alpha)^{\beta}}\right]$$

The integral is numerically evaluated using Gaussian Quadrature. (Abramowitz M., 1972)

The down time is the time needed to repair or replace the component.

$$E\left[\boldsymbol{R}\right] = P\left\{\boldsymbol{T}_W < \boldsymbol{T}_F\right\} r_W + P\left\{\boldsymbol{T}_F \le \boldsymbol{T}_W\right\} r_F$$

$$= P\left\{\boldsymbol{\varphi} < 1\right\} r_W + P\left\{\boldsymbol{\varphi} \ge 1\right\} r_F$$

$$= \left[1 - e^{-\alpha^{\beta}}\right] r_W + e^{-\alpha^{\beta}} r_F$$

From renewal reward process theory the long run average time the system is down is

$$\frac{E[\boldsymbol{R}]}{E[\boldsymbol{R}]+E\left[\min\left(\boldsymbol{T}_W,\boldsymbol{T}_F\right)\right]}$$

The long run average time the system is up is

$$\frac{E\left[\min\left(\boldsymbol{T}_W,\boldsymbol{T}_F\right)\right]}{E[\boldsymbol{R}]+E\left[\min\left(\boldsymbol{T}_W,\boldsymbol{T}_F\right)\right]}$$

APPENDIX D: SCHEDULED MAINTENANCE MODEL

The failure of the component is not directly observable. The component must be inspected to determine failure, there is not a sensor. For this model, the component is inspected periodically. Each inspection of the component takes r_I time units; each repair/replacement of a failed component takes r_F time units. When the component fails it is down until the next inspection.

Since the failure times are exponentially distributed, a cycle begins and ends at the end of each inspection and subsequent repair if needed. The expected time the component is down in a cycle is

$$r_I + r_F\left[1-e^{-\lambda T}\right] + \int_0^T [T-s]\lambda e^{-\lambda s} ds$$
$$= r_I + r_F\left[1-e^{-\lambda T}\right] + T - \frac{1}{\lambda}\left[1-e^{-\lambda T}\right]$$

The expected time the component is up during a cycle is

$$E[\boldsymbol{T}_F] = \int_0^T s\lambda e^{-\lambda s} ds + Te^{-\lambda T} = \frac{1}{\lambda}\left[1-e^{-\lambda T}\right]$$

The long run average time the system is up (or operational availability) is

$$\frac{E[Up\,Time]}{E[Cycle\,Time]} = \frac{\frac{1}{\lambda}\left[1-e^{-\lambda T}\right]}{T + r_I + r_F\left[1-e^{-\lambda T}\right]}$$

To find the inter-inspection time, T, that maximizes the operational availability take the derivative of the concave function that defines the operational availability with respect to T and set that equal to zero.

$$\frac{d}{dT}\frac{E[\boldsymbol{D}]}{E\big[\boldsymbol{C}\big]} = \frac{r_F(\lambda e^{-\lambda T}) + 1 - e^{-\lambda T}}{T + r_I + r_F(1 - e^{-\lambda T})} - \frac{\left[r_I + r_F(1 - e^{-\lambda T}) + T - \frac{1}{\lambda}(1 - e^{-\lambda T})\right]}{(T + r_I + r_F(1 - e^{-\lambda T}))^2}\left[1 + r_F\lambda e^{-\lambda T}\right] = 0$$

$$\Rightarrow$$

$$\left[r_F(\lambda e^{-\lambda T}) + 1 - e^{-\lambda T}\right]\left[T + r_I + r_F(1 - e^{-\lambda T})\right] = \left[r_I + r_F(1 - e^{-\lambda T}) + T - \frac{1}{\lambda}(1 - e^{-\lambda T})\right]\left[1 + r_F\lambda e^{-\lambda T}\right]$$

Solve for T numerically. In this thesis, the maximizing value of T is found by evaluating the operational availability using a search over the time interval from 0 to 300 with grid of one time unit as well as using Microsoft Excel solver.

Using a Taylor series expansion where $e^{-\lambda T} \approx 1 - \lambda T$ gives the following approximation (note: generally the Taylor series expansion approximation is better when λT is close to zero):

$$\left[r_F\lambda(1-\lambda T)+\lambda T\right]\left[T+r_I+r_F\lambda T\right]=\left[r_I+r_F\lambda T+T-\frac{\lambda}{\lambda}T\right]\left[1+r_F\lambda(1-\lambda T)\right]$$

$$\begin{aligned}
&LHS:\\
&\left[r_F\lambda-r_F\lambda^2T+\lambda T\right]\left[T+r_I+r_F\lambda T\right]=\\
&r_F\lambda T+r_F\lambda r_I+r_F^2\lambda^2T-\left[r_F\lambda^2T^2+r_F\lambda^2Tr_I+r_F^2\lambda^3T^2\right]+\lambda T^2+\lambda Tr_I+r_F\lambda^2T^2\\
&=r_F\lambda r_I+T\left(r_F\lambda+r_F^2\lambda^2-r_F\lambda^2r_I+\lambda r_I\right)+T^2\left(\lambda-r_F^2\lambda^3\right)\\
&RHS:\\
&\left[r_I+r_F\lambda T\right]\left[1+r_F\lambda-r_F\lambda^2T\right]\\
&=r_I+r_F\lambda T+r_Ir_F\lambda+r_F^2\lambda^2T-r_Ir_F\lambda^2T-r_F^2\lambda^3T^2\\
&=r_I+r_Ir_F\lambda+T\left(r_F\lambda+r_F^2\lambda^2-r_Ir_F\lambda^2\right)-T^2\left(r_F^2\lambda^3\right)\\
&\Rightarrow
\end{aligned}$$

$$\begin{aligned}
&r_F\lambda r_I+T\left(r_F\lambda+r_F^2\lambda^2-r_F\lambda^2r_I+\lambda r_I\right)+T^2\left(\lambda-r_F^2\lambda^3\right)=\\
&r_I+r_Ir_F\lambda+T\left(r_F\lambda+r_F^2\lambda^2-r_Ir_F\lambda^2\right)-T^2\left(r_F^2\lambda^3\right)
\end{aligned}$$

$$\begin{aligned}
&Combining\ Terms:\\
&-r_I+T\left(\lambda r_I\right)+T^2\left(\lambda\right)=0
\end{aligned}$$

$$\begin{aligned}
&Quadratic\ Equation:\\
&T=\frac{-\lambda r_I\pm\sqrt{\left(\lambda r_I\right)^2-4(-r_I)(\lambda)}}{2\lambda}=\frac{-\lambda r_I\pm\sqrt{\left(\lambda r_I\right)^2+4\lambda r_I}}{2\lambda}
\end{aligned}$$

The approximation to the maximizing inter-inspection time is the positive root. Table 28 displays results from this approximation and the maximizing inter-inspection time. The mean repair time due to failure is $r_F = 2$ for all cases. The results are displayed graphically in Figure 14. Due to the flat nature of the availability curve, the approximations for the optimal T are consistently lower than the optimal inter-inspection time. However the operational availability resulting from the two inter-inspection times are close.

For $r_F = 2$	Numerical Result	Approximation
$r_I = 1$	14	9.5
$r_I = 2$	19	13.2
$r_I = 3$	24	15.9
$r_I = 4$	27	18.1

Table 28. Optimal and Approximately Optimal Inspection Intervals for Given r_I

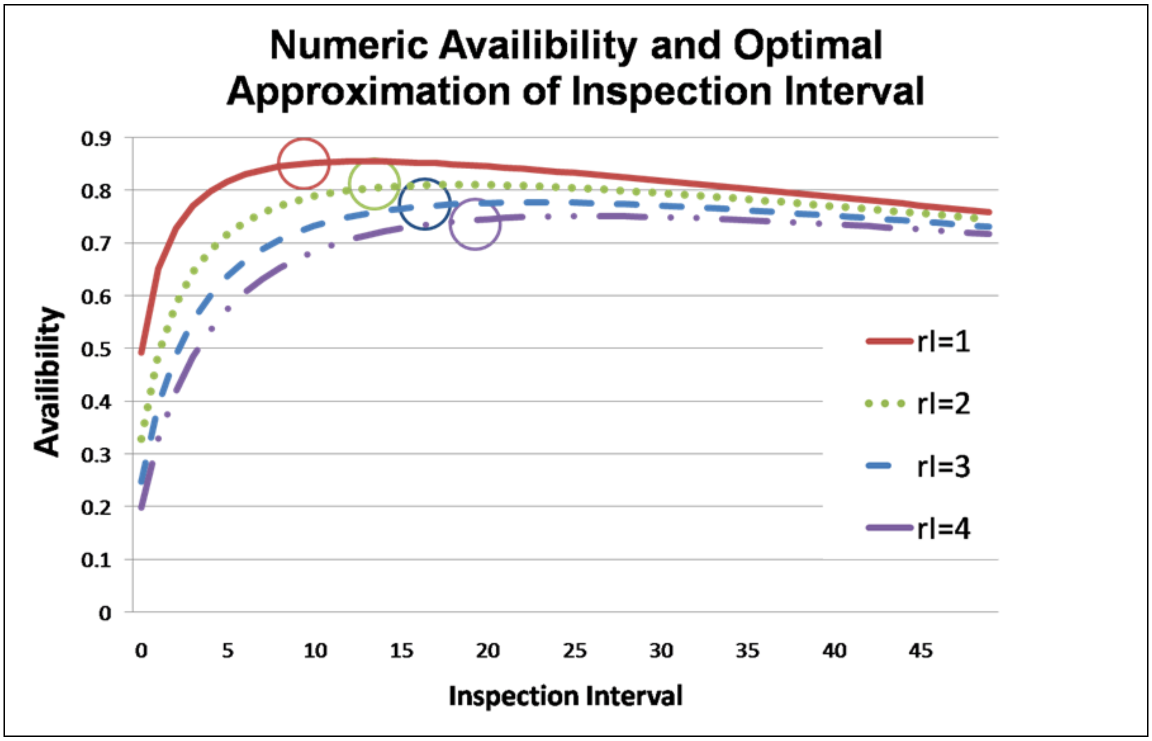

Figure 14. Plot of Availability for Given Inspection Intervals; with circles surrounding the approximate maximizing inter-inspection interval.

APPENDIX E: SENSOR & INSPECTION BASED PREDICTIVE MAINTENANCE MODEL

The failure of the component is not observable directly but only through inspection. The component has a sensor which may provide warning of failure. There is also an inspection at fixed intervals of time, T, after each repair/replacement of the component. The component is repaired to as good as new whenever a warning occurs or when the system is found failed during an inspection. Again, if the component fails without warning, then it is down until its next inspection. A cycle here begins and ends each time there is a warning or an inspection reveals a failure and the component is repaired

The expected cycle time is

$$E\left[\textbf{\textit{Cycle Time}}\right]=P\{\boldsymbol{\varphi}\ge 1\}\left[\sum_{n=1}^{\infty}\left(e^{-\lambda T}\right)^{n-1}\left[1-e^{-\lambda T}\right]n\left[r_I+T\right]+r_F\right]$$

$$+P\{\boldsymbol{\varphi}<1\}\,E\left[\sum_{n=1}^{\infty}\left(e^{-(\lambda/\boldsymbol{\varphi})T}\right)^{n-1}\left[1-e^{-(\lambda/\boldsymbol{\varphi})T}\right](n-1)r_I+r_W+\boldsymbol{T}_W\mid\boldsymbol{\varphi}<1\right]$$

$$=P\{\boldsymbol{\varphi}\ge 1\}\left[\frac{1}{1-e^{-\lambda T}}\left[r_I+T\right]+r_F\right]+P\{\boldsymbol{\varphi}<1\}\,E\left[\left[\frac{e^{-(\lambda/\boldsymbol{\varphi})T}}{1-e^{-(\lambda/\boldsymbol{\varphi})T}}r_I+r_W+\boldsymbol{\varphi}\boldsymbol{T}_F\right]\mid\boldsymbol{\varphi}<1\right]$$

$$=P\{\boldsymbol{\varphi}\ge 1\}\left[\frac{1}{1-e^{-\lambda T}}\left[r_I+T\right]+r_F\right]$$

$$+P\{\boldsymbol{\varphi}<1\}\,r_W+r_I\int_0^1\frac{1}{e^{(\lambda/y)T}-1}G(dy)+E\left[\boldsymbol{T}_F\right]\int_0^1 yG(dy)$$

where G is the distribution function of a Weibull random variable;

$$G(dy)=g(y)\,dy=dG(y)=\alpha^{\beta}\beta y^{\beta-1}e^{-(\alpha y)^{\beta}}dy$$

The expected down time is

$$E\left[\boldsymbol{D}\right]=P\left\{\boldsymbol{\varphi}>1\right\}\left[\sum_{n=1}^{\infty}\left(e^{-\lambda T}\right)^{n-1}\left[1-e^{-\lambda T}\right]nr_I+r_F+\int_0^T\left(T-s\right)\lambda e^{-\lambda s}ds\right]$$
$$+E\left[\sum_{n=1}^{\infty}\left(e^{-(\lambda/\boldsymbol{\varphi})T}\right)^{n-1}\left[1-e^{-(\lambda/\boldsymbol{\varphi})T}\right]\left(n-1\right)r_I+r_W;\boldsymbol{\varphi}<1\right]$$
$$=P\left\{\boldsymbol{\varphi}\geq 1\right\}\left[\left[\frac{1}{1-e^{-\lambda T}}\right]r_I+r_F+T-\frac{1}{\lambda}\left[1-e^{-\lambda T}\right]\right]+P\left\{\boldsymbol{\varphi}<1\right\}E\left[\left[\frac{e^{-(\lambda/\boldsymbol{\varphi})T}}{1-e^{-(\lambda/\boldsymbol{\varphi})T}}r_I+r_W\right]|\,\boldsymbol{\varphi}<1\right]$$
$$=P\left\{\boldsymbol{\varphi}>1\right\}\left[\left[\frac{1}{1-e^{-\lambda T}}\right]r_I+r_F+T-\frac{1}{\lambda}\left[1-e^{-\lambda T}\right]\right]$$
$$+r_W P\left\{\boldsymbol{\varphi}<1\right\}+r_I\int_0^1\frac{1}{e^{(\lambda/y)T}-1}G\left(dy\right)$$

The expected up time is

$$E\left[\boldsymbol{U}\right]=P\left\{\boldsymbol{\varphi}>1\right\}E\left[\boldsymbol{T}_F\right]+E\left[\boldsymbol{T}_W;\boldsymbol{\varphi}<1\right]$$
$$=P\left\{\boldsymbol{\varphi}>1\right\}E\left[\boldsymbol{T}_F\right]+E\left[\boldsymbol{T}_F\right]E\left[\boldsymbol{\varphi};\boldsymbol{\varphi}<1\right]$$
$$=E\left[\boldsymbol{T}_F\right]\left[\int_0^1 yG\left(dy\right)+P\left\{\boldsymbol{\varphi}>1\right\}\right]$$
$$=E\left[\boldsymbol{T}_F\right]E\left[\min\left(\boldsymbol{\varphi},1\right)\right]$$

Then the long run proportion of time the component is up is

$$\frac{E\left[\boldsymbol{U}\right]}{E\left[\boldsymbol{C}\right]}$$

APPENDIX F: IMPACT OF SENSOR EFFECTIVENESS ON MISSION TIMES AND AVAILABILITY

The following describes the simulation for effect of the sensor warnings on reducing component failure during a mission. For given Weibull parameters consideration is given to the fraction of missions having a warning that also have a failure. Additionally, the average number of missions that would follow the warning until failure occurs is determined. Each simulation has 10,000 replications. A description of one replication of the simulation is:

Specify a mission time m

Draw exponential failure time, $\boldsymbol{T}_F$

Draw Weibull random variable, $\boldsymbol{\varphi}$, if $\boldsymbol{\varphi} > 1$ then there is no warning and $\boldsymbol{\varphi}$ is redrawn until $\boldsymbol{\varphi} < 1$ (only replications with warning before failure are considered).

When $\boldsymbol{\varphi} < 1$ do the following

Calculate the warning time, $\boldsymbol{T}_W$

Let $N_W = \left\lfloor \frac{\boldsymbol{T}_W}{m} \right\rfloor =$ number of missions without warning; warning will occur on mission $N_W + 1$.

Let $N_F = \left\lfloor \frac{\boldsymbol{T}_F}{m} \right\rfloor =$ number of missions without failure; failure will occur on mission $N_F + 1$

If $N_F = N_W$ record a failure during the mission in which the warning occurred.

The simulation is replicated 10,000 times.

The fraction of missions with a warning and a failure $= \frac{\# of\ times\ N_F = N_W}{10,000}$

The number of additional successful missions without failure is computed in the following manner:

$0\ if\ \ N_W = N_F$
$0\ if\ \ N_W + 1 = N_F$
$N_F - (N_W + 1)\ otherwise$

APPENDIX G: ADDITIONAL COMPARISON OF ANALYTICAL RESULTS

Further results of Table 8 Operational Availability from Chapter V Section B are displayed here.

	Constant φ	φ having a Weibull Distribution	Constant φ	φ having a Weibull Distribution
Inspection Time T	P{$\varphi = .9$}=1	Median =0.9 and Shape Parameter = 30	P{$\varphi = .5$}=1	Median =0.5 and Shape Parameter = 5
1	0.49587	0.49584	0.49260	0.49251
2	0.65933	0.65929	0.65357	0.65341
3	0.74072	0.74067	0.73344	0.73324
4	0.78945	0.78938	0.78117	0.78094
5	0.82188	0.82181	0.81290	0.81265
6	0.84503	0.84495	0.83551	0.83525
7	0.86237	0.86229	0.85245	0.85217
8	0.87585	0.87577	0.86560	0.86531
9	0.88663	0.88654	0.87611	0.87581
10	0.89544	0.89535	0.88469	0.88439
11	0.90278	0.90269	0.89184	0.89152
12	0.90899	0.90890	0.89788	0.89756
13	0.91431	0.91422	0.90305	0.90272
14	0.91892	0.91882	0.90753	0.90719
15	0.92295	0.92285	0.91144	0.91110
16	0.92650	0.92641	0.91489	0.91454
17	0.92966	0.92956	0.91795	0.91759
18	0.93248	0.93239	0.92068	0.92032
19	0.93502	0.93493	0.92314	0.92278
20	0.93732	0.93722	0.92536	0.92499
21	0.93941	0.93931	0.92737	0.92700
22	0.94131	0.94122	0.92921	0.92884
23	0.94306	0.94296	0.93089	0.93051
24	0.94466	0.94456	0.93243	0.93205

25	0.94614	0.94604	0.93386	0.93347
26	0.94751	0.94741	0.93517	0.93478
27	0.94878	0.94869	0.93639	0.93600
28	0.94997	0.94987	0.93752	0.93713
29	0.95107	0.95097	0.93858	0.93818
30	0.95210	0.95200	0.93956	0.93916
31	0.95307	0.95297	0.94048	0.94008
32	0.95398	0.95388	0.94135	0.94094
33	0.95483	0.95473	0.94216	0.94175
34	0.95563	0.95553	0.94292	0.94251
35	0.95639	0.95629	0.94364	0.94322
36	0.95711	0.95701	0.94432	0.94390
37	0.95779	0.95769	0.94496	0.94453
38	0.95843	0.95833	0.94556	0.94514
39	0.95904	0.95894	0.94614	0.94571
40	0.95962	0.95952	0.94668	0.94625
41	0.96017	0.96007	0.94720	0.94677
42	0.96070	0.96060	0.94769	0.94726
43	0.96120	0.96110	0.94816	0.94772
44	0.96168	0.96158	0.94861	0.94817
45	0.96214	0.96204	0.94903	0.94859
46	0.96258	0.96247	0.94944	0.94900
47	0.96300	0.96289	0.94983	0.94938
48	0.96340	0.96329	0.95020	0.94975
49	0.96378	0.96368	0.95056	0.95010
50	0.96415	0.96405	0.95090	0.95044

Table 29. Additional Comparison of Analytical Results

APPENDIX H: VBA CODE

```
'File Name:  All_in_One
'
'Created: April May 2009 ( Naval Postgraduate School)
'This program is used to calculate availability and long run average cost of
'different maintenance policies given certain inputs.
'
'Author:  Maj Pete Koeneman with some code written by Professor P.A. Jacobs (Thank You!)
'
'  Some input variables used throughout the program
Public alpha As Double                'Weibull scale parameter
Public beta As Double                 'Weibull shape parameter
Public cweib As Double                'Frequently used constant
Public lambda As Double               'Exponential parameter for failure rate
Public InspTime As Double             'Inspection Time interval
Public TTC As Double                  'Time to Consider for simulation
Public rI As Double                   'Expected time to complete Inspection
Public rW As Double                   'Expected time to complete repair due to Warning
Public rF As Double                   'Expected time to complete repair due to Failure
Public cI As Double                   'Expected cost to complete Inspection
Public cW As Double                   'Expected cost to complete repair due to Warning
Public cF As Double                   'Expected cost to complete repair due to Failure
Public Mt As Double                   'Mission Time
Public IntegralyDens01 As Double      'Value of a Integral from integral()
Public IntegralDens02 As Double       'Value of a Integral from integral2()


'The main subroutine that reads in variables and then computes values or calls other
'subroutines to compute values and then displays them out to Output sheet.

Sub Main()
'clear cells for new output produced
Worksheets("Output").Range("E5:AA1000").ClearContents
Worksheets("DataOut").Range("c11:CA1000").ClearContents
'Fix time and cost of warnings to two
rW = 2
cW = 2
'For each of the alpha beta parameter combinations for the Weibull Random Variable
For paracount = 0 To 17
   beta = (Worksheets("Inputs").Cells(70 + paracount, 1))
   alpha = (Worksheets("Inputs").Cells(70 + paracount, 2))
   'For inspection time and costs 1 thru 4
```

```
For ccr = 0 To 3
   rI = 1 + ccr
   cI = 1 + ccr

      'For failure time and costs 2 thru 7
      For ccc = 0 To 5

         cF = 2 + ccc
         rF = 2 + ccc
         'Read in values for program variables
         Worksheets("Inputs").Cells(2, 2) = alpha
         Worksheets("Inputs").Cells(3, 2) = beta
         lambda = (Worksheets("Inputs").Cells(4, 2))
         TTC = (Worksheets("Inputs").Cells(6, 2))
         Worksheets("Inputs").Cells(9, 2) = rI
         Worksheets("Inputs").Cells(10, 2) = rW
         Worksheets("Inputs").Cells(11, 2) = rF
         Worksheets("Inputs").Cells(12, 2) = cI
         Worksheets("Inputs").Cells(13, 2) = cW
         Worksheets("Inputs").Cells(14, 2) = cF
         InspTime = (Worksheets("Inputs").Cells(16, 2))

         'ONE   Run to Failure Maintenance Policy Calculate % up time and cost
         'if there are no inspections or warnings only failures
         noIW = 1 - (rF / (rF + (1 / lambda)))
         'Calculate long run average cost of no inspection or warnings
         cnoIW = cF / (rF + (1 / lambda))


         'Call integral subroutine
         Call integral


         'TWO   Calculate analytical % up time with warnings and failures only
         pphigtone = Exp(-(alpha ^ beta))    'Probability phi is greater than one
         Worksheets("Output").Cells(5, 2) = pphigtone
         'Calculate the Median of the weibull random variable phi
         Worksheets("Output").Cells(6, 2) = (1 / alpha) * (Log(2)) ^ (1 / beta)
         ER = (rW * (1 - pphigtone)) + (rF * (pphigtone)) 'Expected repair time

         WnoI = 1 - (ER / (ER + ((1 / lambda) * (Worksheets("Output").Cells(2, 2)))))
         cWnoI = cW * (1 - pphigtone) + cF * (pphigtone)
         'Calculate the Long Run Average Cost of warning and failures
         cWnoI2 = (cWnoI / (ER + (1 / lambda) * Worksheets("Output").Cells(2, 2)))
```

```
'Loop thru the requested values of InspTime & apply to equations that are functions of InspTime
Dim count As Integer
count = TTC / InspTime
Worksheets("Output").Cells(4, 2) = count

For j = 0 To count
   InspTime = (j + 1) * (Worksheets("Inputs").Cells(16, 2))
   'Call integral2 subroutine
   Call integral2

   'THREE
   'Calculate analytical % up time with Inspection only(a function of InspTime)
   Ionly = ((1 / lambda) * (1 - Exp(-lambda * InspTime)) / (InspTime + rI + rF * (1 - Exp(-lambda * InspTime))))
   cIonly = cF + ((1 / lambda) / InspTime) * cI
   EcIonly = InspTime + rI + rF * (1 - Exp(-(lambda * InspTime)))
   cIonly2 = cI + cF * (1 - Exp(-lambda * InspTime))

   'FOUR
   'Calculate analytical % up time with both warning and inspections (a function of InspTime)
   Eu = 100 * Worksheets("Output").Cells(2, 2)
   Ec = (pphigtone * (rF + (rI + InspTime) * (1 / (1 - Exp(-lambda * InspTime))))) + ((1 - pphigtone) * rW) + (rI * IntegralDens02) + ((1 / lambda) * IntegralyDens01)
   Ed = (pphigtone * (rF + InspTime - ((1 / lambda) * (1 - Exp(-lambda * InspTime))) + rI * (1 / (1 - Exp(-lambda * InspTime))))) + ((1 - pphigtone) * rW) + (rI * IntegralDens02)
   numbInsp = Ec / InspTime
   cWandI = (cW * (1 - pphigtone) + cF * (pphigtone) + (cI * IntegralDens02) + (cI * pphigtone / (1 - Exp(-lambda * InspTime)))) / Ec

   'print to the Output sheet the percentage up times and costs for the different maintenance methods
   Worksheets("Output").Cells(5 + j, 5) = InspTime
   Worksheets("Output").Cells(5 + j, 12) = cF
   Worksheets("Output").Cells(5 + j, 6) = noIW
   Worksheets("Output").Cells(5 + j, 13) = cWnoI
   Worksheets("Output").Cells(5 + j, 7) = WnoI
   Worksheets("Output").Cells(5 + j, 8) = Ionly
   Worksheets("Output").Cells(5 + j, 14) = cIonly
   Worksheets("Output").Cells(5 + j, 9) = Eu / Ec
   Worksheets("Output").Cells(5 + j, 15) = cWandI
```

```
            Worksheets("Output").Cells(5 + j, 16) = IntegralDens02
            Worksheets("Output").Cells(5 + j, 17) = cIonly2 / EcIonly
            Worksheets("Output").Cells(5 + j, 18) = cWnoI2
            Worksheets("Output").Cells(5 + j, 19) = cnoIW
        Next j
      'Copy and paste values to set up a table of results
      Worksheets("Inputs").Range("B19:D22").Copy
      Worksheets("DataOuts").Cells((11 + 4 * paracount), (4 + (24 * ccr) + (3 *
ccc))).PasteSpecial (xlPasteValues)
      'Create a table of comparisons for different maintenance methods
      Worksheets("Inputs").Cells(19, 5).Copy
      Worksheets("Comparison").Cells((11 + paracount), (1 + 6 * ccr +
ccc)).PasteSpecial (xlPasteValues)
      Worksheets("Inputs").Cells(22, 5).Copy
      Worksheets("Comparison").Cells((31 + paracount), (1 + 6 * ccr +
ccc)).PasteSpecial (xlPasteValues)
      Worksheets("Inputs").Cells(19, 6).Copy
      Worksheets("Comparison").Cells((51 + paracount), (1 + 6 * ccr +
ccc)).PasteSpecial (xlPasteValues)
      Worksheets("Inputs").Cells(22, 6).Copy
      Worksheets("Comparison").Cells((71 + paracount), (1 + 6 * ccr +
ccc)).PasteSpecial (xlPasteValues)



    Next ccc
  Next ccr
Next paracount


End Sub

'========================================

Sub integral()
'From Professor P.A. Jacobs (Naval Postgraduate School)
'Uses gaussian quadrature with weights and t values from Abramowitz and Stegun
'Handbook of Mathematical Functions to evaluate integral for a function listed

iin = "gauss24"
numb = 24
''''''''''''
alpha = Worksheets("Inputs").Cells(2, 2)
beta = Worksheets("Inputs").Cells(3, 2)
cweib = (alpha ^ beta) * beta
```

```
lowb = 0
highb = 1
mm = 0.5 * (highb - lowb)
cc = 0.5 * (highb + lowb)
sumden = 0
sumnum = 0
sumcheck = 0
For i = 1 To numb
T = Worksheets(iin).Cells(i + 1, 1)
w = Worksheets(iin).Cells(i + 1, 2)
y = cc + (mm * T)
ffcheck = y
ff1num = (y ^ beta)
ff2num = Exp(-((alpha * y) ^ beta))
ffnum = ff1num * ff2num
sumnum = sumnum + (ffnum * w)
sumcheck = sumcheck + (ffcheck * w)
Next i
intnum = mm * sumnum
intcheck = mm * sumcheck
IntegralyDens01 = intnum * cweib
term = Exp(-(alpha * 1) ^ beta)
expected = (intnum * cweib) + term
'Expected value (min weibull RV , 1)
Worksheets("Output").Cells(2, 2) = expected
Worksheets("Output").Cells(1, 2) = IntegralyDens01

End Sub

'======================================

Sub integral2()
'From Professor P.A. Jacobs (Naval Postgraduate School)
'Uses gaussian quadrature with weights and t values from Abramowitz and Stegun
'Handbook of Mathematical Functions to evaluate integral for a function listed
iin = "gauss24"
numb = 24
''''''''''
'alpha = Worksheets("Inputs").Cells(2, 2)
'beta = Worksheets("Inputs").Cells(3, 2)
cweib = (alpha ^ beta) * beta
lowb = 0
highb = 1
mm = 0.5 * (highb - lowb)
cc = 0.5 * (highb + lowb)
```

```
sumden = 0
sumnum = 0
sumcheck = 0
For i = 1 To numb
T = Worksheets(iin).Cells(i + 1, 1)
w = Worksheets(iin).Cells(i + 1, 2)
y = cc + (mm * T)
ffcheck = y
ff1num = (y ^ (beta - 1))
ff2num = Exp(-((alpha * y) ^ beta))
ff3num = (Exp(-(lambda * InspTime) / y)) / (1 - Exp(-(lambda * InspTime) / y))
ffnum = ff1num * ff2num * ff3num
sumnum = sumnum + (ffnum * w)
sumcheck = sumcheck + (ffcheck * w)
Next i
intnum = mm * sumnum
intcheck = mm * sumcheck
Worksheets("Output").Cells(3, 2) = intnum * cweib
IntegralDens02 = intnum * cweib

End Sub
```

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